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A mathematical Model for Cadmium Removal using A sulfate Reducing Bacterium: *Desulfovibrio alaskensis* 6SR

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ABSTRACT: In this work, an unstructured-type mathematical model was developed to simulate cadmium (Cd^{2+}) removal by *Desulfovibrio alaskensis* 6SR, which is a recently described sulfate reducing bacterium, whose capacity for removing heavy metals are being studied. Three processes are considered in the model: 1) the sulfate reduction process, 2) the consumption of lactate as carbon source, and 3) the metal removal. The model was tested with different initial Cd^{2+} concentrations (50, 100, 170, and 190 mg/L), and it accurately predicted the behavior of experimental data with satisfactory correlation coefficients (R^2>0.97). In addition, the model showed that the H_2S production rate and initial concentration of Cd^{2+} are key operating variables in a bioreactor. *Desulfovibrio alaskensis* 6SR was able to remove more than 99.9% of cadmium in a batch process, where the initial concentration was 170 mg/L. The model, applied to a continuous process, predicted a maximum Cd^{2+} removal of 99.1% with the same initial concentration. Also, the model predicted the inhibitory effect of initial Cd^{2+} concentrations above 190 mg/L. The mathematical model developed can be used for optimization and control purposes.

Key words: Desulfovibrio alaskensis, Dynamic, Modeling, Cadmium removal, Kinetic, Anaerobic

INTRODUCTION

Sulfate reducing bacteria (SRB) form a group of prokaryotic organisms able to transform sulfate into hydrogen sulfide (H_2S). Members of the SRB are widespread in anoxic habitats, where they play pivotal roles in the sulfur and carbon cycles (Castro *et al.,* 2000). The high reactivity of hydrogen sulfide induces corrosion of metals, causing substantial economic losses in the petroleum industry and other industries (Johnson *et al.,* 2002; Boddu *et al.,* 2003; Videla *et al.,* 2005). Conversely, SRB have been used to solve a number of environmental problems, e.g., removal of metals from wastewater through the production of biogenic sulfides, followed by metal precipitation (Quintelas *et al.,* 2001; Muyzer *et al.,* 2008; Natarajan, 2008). This is illustrated in the following reactions:

$$SO_4^{2-} + 8e^- + 4H_2O \rightarrow S^{2-} + 8OH^- \quad (1)$$

$$S^{2-} + M^{2+} \rightarrow MS\downarrow \quad (2)$$

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Species of the genus *Desulfovibrio* are the most studied in this field, showing a high efficiency in removing different metallic ions in a range from a few ppm (mg/L) to as much as 100 mg/L, Zn^{2+} (25-40 mg/L), Pb^{2+} (75-80 mg/L), Cd^{2+} (4-20 mg/L), and Cr^{6+} (60 mg/L) (Utgikar, 2002; Naz *et al.,* 2005; Cabrera *et al.,* 2006). Many experimental studies have been conducted with SRB for heavy metals removal from wastewater effluents (Sheng *et al.,* 2010; Volesky, 2007; Volesky *et al.,* 2005). However, few published works have reported the mathematical modeling of metal removal by bio-precipitation (see Table 1). Several mathematical models have been developed for natural systems, some include affinity models such as the Monod-type, others are based on product or substrate inhibition, such as those developed by Haldane, Andrew's, Luong's, Han-Levenspiel, Boulton, Moser and Aiba (Okpokwasili *et al.,* 2005; Han *et al.,* 1988; Luong *et al.,* 1987). All these models describe the relationship between bacterial specific growth rate (μ) and

Table 1. Unstructured models developed for sulfate-reducing systems

Unstructured models	Studied variables	Reference
Michaelis-Menten and Fick's	Substrate diffusion in the biofilm is described by Fick's second law.	Nielsen *et al.,* (1987)
Monod	Inhibition by undissociated H_2S and pH.	Kalyuzhnyi *et al.,* (1998)
Sulfate reduction, zero-order	Cadmium precipitation	White *et al.,* (2003)
Michaelis–Menten	Microbial sulfate reduction.	Pallud *et al.,* (2006)
Modified Monod	Considers pH and sulfide concentration as inhibitory factors	Torner-Morales *et al.,* (2010)
Non-competitive inhibition	Thermophilic bacteria with L-lactate	Hidaka *et al.,* (2010)

substrate concentration. In other cases, combinations of two models were used: Haldane-Boulton, Haldane-Levenspiel, Haldane-Luong, Moser-Boulton, Moser-Levenspiel, and Moser-Luong models (Monod *et al.,* 1949). Moosa *et al.* (2002) reported the effects of sulfate concentration and temperature on bacterial growth rate. Their experimental data were fitted with different mathematical models including those of Monod, Chen and Hashimoto, and Contoins (Moosa *et al.,* 2005). However, more widely applicable models are not available, e.g., to describe the combined effects of sulfate, hydrogen sulfide, biomass, lactate, acetate, carbon dioxide, on the removal of Cd^{2+} in anaerobic processes.

In this work, batch cultures of *Desulfovibrio alaskensis* 6SR with lactate as carbon source were incubated to evaluate the kinetics of Cd^{2+} removal at different initial concentrations (50, 100, 170, and 190 mg/L). The effect of Cd^{2+} concentration on bacterial growth was also evaluated. A mathematical model is proposed which considers unstructured-type kinetics, including the kinetics of bacterial growth, sulfate and lactate consumption rates, inhibition by hydrogen sulfide, production of acetate and carbon dioxide (CO_2), and Cd^{2+} removal. The model parameters were fitted with data obtained from experiments performed with initial Cd^{2+} concentrations below 170 mg/L. As expected, the model is able to closely predict the time course of Cd^{2+} removal regardless of the initial cadmium concentration. In order to determine acceptable operating conditions for Cd^{2+} removal, the model was used to compare continuous and batch processes via numerical simulations.

MATERIALS & METHODS

The bacterium *Desulfovibrio alaskensis* was described firstly by Feio *et al.* (2004), subsequent studies related to bio-corrosion reported that *D. alaskensis* is present in oil fields from the Gulf of Mexico (Hernández-Gayoso *et al.,* 2004). The strain *D. alaskensis* 6SR was isolated from a biofilm developed on the inside face of an oil pipeline. *Desulfovibrio alaskensis* 6SR was isolated from biofilm samples. To obtain the biofilm samples, an API X52 steel coupon (surface area of 4 cm^2) was placed at the inner surface of two different pipelines made of the same material and exposed for 40 days to the flow of petroleum. The biofilm samples were used directly to culture and isolate sulfate-reducing bacteria. Sequence analysis of 16S rRNA gene (DQ083981) of the 6SR strain revealed that it has a 99.8% similarity with *Desulfovibrio alaskensis* (Neria-González *et al.,* 2006). The bacterium was grown in Postgate C medium with sodium sulfate and lactate as carbon source (Postgate *et al.,* 1986). All reagents were analytical grade (Sigma-Aldrich®). The medium's pH was adjusted to 7.0 with sterile NaOH solution (0.5 M). The medium was prepared and handled under a N_2 atmosphere (99.998% purity). Serum bottles (120 mL) were filled with 45 mL of the medium and sterilized (121ºC, 15 min). A stock cadmium acetate solution was prepared with 10,000 mg/L of Cd^{2+}. The solution was sterilized by membrane ûltration (pore size 0.22 µm). The serum bottles with the culture medium were spiked with the stock solution to obtain 50, 100, 170, and 190 mg/L of Cd^{2+}. The inoculum was prepared in 500 mL of modified Postgate's C medium at 37°C until an OD_{580} between 0.35 and 0.4 was obtained. A 5-mL-aliquot was taken to inoculate the bottles containing 45 mL of fresh medium with cadmium. Each experiment with a given metal concentration consisted of 10 bottles (and 10 duplicates). In addition, five cultures were performed without cadmium (controls). The batch cultures were performed under anaerobic conditions at 37°C for 216 hours. A bottle was taken every 24 hours from each series with different initial Cd^{2+} concentration, for analyses of; sulfate, sulfide, biomass, lactate, acetate, carbon dioxide, and cadmium concentration.

A colorimetric assay method was used for hydrogen sulfide analyses (Cord-Ruwisch *et al.,* 1985). The bacterial biomass growth was evaluated by the dry weight method (APHA *et al.,* 1985). Sulfates were assessed by the turbidimetric method based on barium precipitation. Lactate and acetate concentrations were evaluated by HPLC in a Shimadzu LC10Ai chromatograph with UV detector ($\lambda = 210$ nm) and a BioRad HPLC Organic Acid Analysis Column. Elution was made at 0.7 ml min^{-1} flow rate, using a solution of sulfuric acid/water (0.33/0.67) as mobile phase. Lactic acid (60% v/v Sigma-Aldrich) and sodium acetate (99.0% Sigma-Aldrich*)* were used as standards. An automated gas chromatographic (GC) system was used to measure carbon dioxide in the gas phase, as reported by Moosa *et al.* (2002). A portion of the gas sample was introduced into a gas sampling loop, injected, and analyzed using a thermal conductivity detector (Guiochon and Pommier, 1973; Coleman III *et al.,* 1996). A 40-mL aliquot of broth from each bottle was centrifuged at 13,000 rpm. The supernatant was recovered and filtered through a nitrocellulose membrane, 0.22 μm pore-diameter. The filtered sample was diluted with 2% (v/v) HNO_3 to determine the cadmium content, and the cells pellet was discarded. Cadmium was assayed by atomic absorption spectrophotometry (AAS) (Atomic Absorption Spectrometer Specter AA-20 plus, Varian) using a lamp exclusive for cadmium (Hollow Cathode Lamp Element: Cadmium, Aurora Instruments), (Arakaki *et al.,* 2002). Biotechnological processes can be analyzed with phenomenological models that describe the dynamic behavior of substrate, biomass, and product concentrations. Dochain *et al.,* (2003) developed methodological approaches for modeling batch bioreactors. The process dynamics can be described by the following model:

$$\frac{d\xi(t)}{dt} = K\varphi(\xi(t)) \tag{3}$$

Where

$\xi \in \Re_+^n$	is the vector of concentration
$K \in \Re_+^{n\,x\,m}$	are the corresponding stoichiometric coefficients matrix ($m < n$),
$\varphi_i \in \Re_+^n$	is the vector of reaction rates kinetic term
$t \in \Re_+$	Time

$$\varphi(\xi(t)) = \mu(t)X(t) \tag{4}$$

Here $\mu(t)$ is the specific growth rate, which is a function of the concentrations; sulfate, hydrogen sulfide, biomass, lactate, acetate, metal.

The conceptual model for Cd^{2+} removal by *Desulfovibrio alaskensis 6SR* under anaerobic conditions is shown in Fig. 1. The proposed model considers the inhibitory effect of Cd^{2+} and hydrogen sulfide on microbial growth (Radha *et al.,* 1992; Medírcio *et al.,* 2007). In addition, the model includes three processes: 1) microbial sulfate reduction, 2) carbon source consumption, 3) Cd^{2+} removal. In experimental conditions, these three processes were monitored, respectively, a) through sulfate, hydrogen sulfide and biomass determinations, b) through lactate, carbon dioxide and acetate analysis, and c) by Cd^{2+} analyses in the supernatant.

A mathematical model was developed to describe the kinetics of Cd^{2+} removal in a batch bioreactor. The classical growth with reduction of sulfate to hydrogen sulfide is described by Equations (5)-(7). The Levenspiel inhibition model was modified to describe the reduction of sulfate. The model considers the inhibitory effect of hydrogen sulfide on microbial growth (Noguera *et al.,* 1998; Al-Tarazi *et al.,* 2004; Cabrera *et al.,* 2006; Van Wageningen *et al.,* 2006).
Sulfate (*S*) mass balance:

$$\frac{dS}{dt} = -\frac{k_{spx}}{Y}\left(1-\frac{P}{k_p}\right)^{\alpha}\left[\frac{S}{k_s+S}\right]X\,L^{\varepsilon} \tag{5}$$

Hydrogen sulfide (*P*) mass balance:

$$\frac{dP}{dt} = \frac{k_{spx}}{Y_p}\left(1-\frac{P}{k_p}\right)^{\alpha}\left[\frac{S}{k_s+S}\right]X\,L^{\varepsilon} \tag{6}$$

Biomass (*X*) balance:

$$\frac{dX}{dt} = k_{spx}\left(1-\frac{P}{k_p}\right)^{\alpha}\left[\frac{S}{k_s+S}\right]\left[\frac{Cdl^{2+}}{k_{Cd}+Cdl^{2+}}\right]^{\eta} X\,L^{\varepsilon} - k_d X\,L^{\varepsilon} \tag{7}$$

A straightforward estimate of acetate and carbon dioxide production, with lactate consumption, was obtained by combination of the Moser-Boulton models and biomass concentration, in accordance with Equations (8) to (10) (Nwabanne *et al.,* 2009; Hidaka *et al.,* 2010):
Lactate (*L*) mass balance:

$$\frac{dL}{dt} = -\frac{k_{LA}}{Y_{L/X}}\left[\frac{k_{ace}}{A+k_{ace}}\right]\left[\frac{L^{\delta}}{k_{lac}+L^{\delta}}\right]X \tag{8}$$

Acetate (*A*) mass balance:

$$\frac{dA}{dt} = \frac{k_{LA}}{Y_{A/X}}\left[\frac{k_{ace}}{A + k_{ace}}\right]\left[\frac{L^{\delta}}{k_{lac} + L^{\delta}}\right]X \qquad (9)$$

Carbon dioxide (*I*) mass balance:

$$\frac{dI}{dt} = \frac{k_{co}}{Y_{I/X}}\left[\frac{k_I}{A + k_I}\right]\left[\frac{L^{\delta}}{k_{lac} + L^{\delta}}\right]X \qquad (10)$$

The mass balance describing the removal of Cd^{2+} is given by a modified Levenspiel-Haldane model (Pokrovsky *et al.,* 2010; Nielsen *et al.,* 2005), see Equation (11):

$$\frac{dCdl}{dt} = -k_{u\max} Cdl\left(1 - \frac{P}{k_p}\right)^{\gamma}\left[\frac{Cdl}{k_1 + Cdl + \frac{Cdl^2}{k_2}}\right]X \qquad (11)$$

To evaluate the metal precipitation capacity of SRB, the percentage of precipitated metal (%PM) was defined with the following equation (Dekhil *et al*., 2011):

$$\%PM = \frac{(Cdl_{t=0} - Cdl_{t=t}) \times 100}{Cdl_{t=0}} \qquad (12)$$

Equations (5)-(11) were used for parameter fitting with the following initial conditions: S_0 = 5655 mg/L; P_0 = 29 mg·L^{-1}, X_0: = 20 mg/L, L_0 = 464 mg/L, A_0 = 0.01 mg/L, I_0 = 0.01 mg/L, Cdl_0 = 170 mg/L. The Model-Maker® software (based on Levenberg–Marquardt optimization approach) was used for the non-linear fitting data. Values previously established for some model parameters were used as initial guess values (Nielsen *et al*., 2005). The performance of the proposed mathematical model was statistically evaluated using the dimensionless coefficient of efficiency (Ω), as suggested by Köhne *et al.* (2006).

$$\Omega = \frac{\sum_{i=1}^{N}\left[\Omega(t_i) - Z(t_i)\right]}{\sum_{i=1}^{N}\left|Z(t_i) - Z^*\right|} \qquad (13)$$

Where $\Omega(t_i)$ is the simulated value of the variable at time t_i, $Z(t_i)$is the observed value, and Z^* is the mean value of the observed variable. Ω varies between - ∞ and 1. A positive value of Ω represents an *acceptable* simulation, whereas Ω > 0.5 represents a *good* simulation.

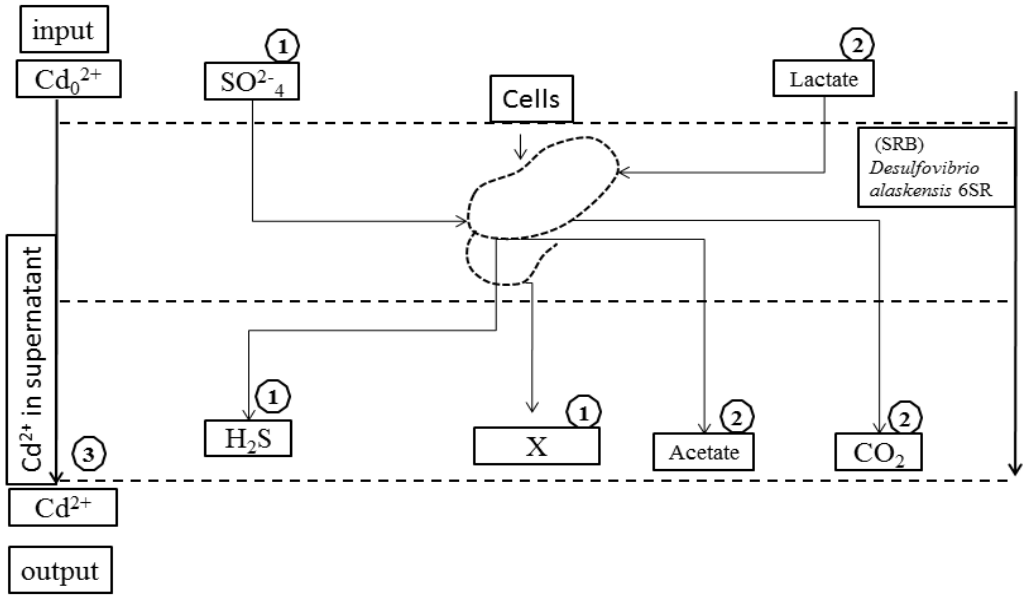

Fig. 1. Conceptual model for Cd^{2+} removal by *Desulfovibrio alaskensis* 6SR under anaerobic conditions

RESULTS & DISCUSSION

Desulfovibrio alaskensis 6SR was used for Cd^{2+} removal under anaerobic conditions. All the experiments were performed with initial biomass concentrations of 20 mg/L, whereas Cd^{2+} concentrations were 50, 100, 170, or 190 mg/L. The initial sulfate and lactate concentrations were 5655 mg/L and 4640 mg/L, respectively. Figs. 2a-2c show the time dependence of sulfate, hydrogen sulfide, and biomass, in the experiments performed with different initial Cd^{2+} concentrations. Figs. 3a-3c, show the corresponding time course of lactate, acetate, and carbon dioxide concentrations, whereas Fig. 4 shows the concentration of Cd^{2+} in solution over time. The Cd^{2+} inhibitory effect on the bacterial growth can be clearly depicted from the diminished end biomass observed at the higher initial Cd^{2+} concentrations (Fig. 2b). A maximum biomass (395 mg/L) was reached with 50 mg/L Cd^{2+}, the lowest concentration, in this case, the maximum sulfate consumption was 89%. On the other hand, the end hydrogen sulfide concentration was between 474 to 713 mg/L depending on the initial Cd^{2+} concentration (see Fig. 2c). Reis *et al*. (2002) and Rivera *et al*. (2003) reported that sulfate transformation to sulfide was inhibited 50% with 480 mg/L hydrogen sulfide while complete inhibition occurred at 547 mg/L. The initial rate (first 24 h) of lactate consumption decreased with increasing Cd^{2+} concentrations as shown in Fig. 3a. Moreover, in our work, Cd^{2+} removal occurred by precipitation of cadmium sulfide. Fig. 4 shows that Cd^{2+} removal was nearly complete within 72 h when the initial Cd^{2+} concentration was below 170 mg/L. The maximum precipitation percentage was 99.9% in all these cases. However, although Cd^{2+} removal also occurred at 190 mg/L initial concentration, it was noticeably incomplete. The relatively low precipitation percentage (82.1%) suggested that this high Cd^{2+} concentration is significantly toxic to *Desulfovibrio alaskensis* (Fig. 5). Considering that maximum metal removal was at 170 mg/L (99.9%), the mathematical model was designed and tuned with this concentration. Similar inhibitory effects and Cd^{2+} removal have been reported by other researchers at similar Cd^{2+} concentrations (Feio *et al.,* 2004; Lewis *et al.,* 2010; Ozturk *et al.,* 2010; Reis *et al.,* 1992; Rivera-Utrilla *et al.,* 2003).

The experimental data served to obtain the 23 parameter values of the model developed (see Table 2). The parametric identification is illustrated by the correlation coefficients above 0.99 between the experimental values and those given by the model (Table 3). The parameter values obtained in the present study are different, this may be due to the different operating conditions used in each case, i.e., metal presence or absence, different carbon source, continuous or batch operation, temperature, pH, among others (see Table 4). The maximum specific growth rate obtained was 0.133 h^{-1}, which is quite similar to that obtained (0.13 h^{-1}) by Feio *et al.* (2004). The average inhibition constant (k_p) determined in this work (680 mg/L) is also in agreement with those reported by others (Table 4). The maximum oxidation of lactate 4640 mg/L was 94%, and the corresponding acetate production was 3790 mg/L, with an experimental error on the stoichiometric balance of 5% (Amor *et al.,* 2001; Nandasana and Kumar, 2008). Finally, the kinetic model was tested considering a continuous operating process with initial Cd^{2+} concentrations of: 50, 100, 170, 190, and 300 mg/L (see Fig. 6). Maximum Cd^{2+} removal (169.1 mg/L with initial Cd^{2+} concentrations under 170 mg/L) was observed at D = 0.01 h^{-1} (residence time 4.1 d) in the continuous operation. The following steady state was reached: (S = 1456.2 mg/L, P = 445.21 mg/L, X = 249.9 mg/L, L = 1457.1 mg/L, A = 2546.3 mg/L, I = 655.6 mg/L, Cdl = 0.69 mg·L^{-1}). The corresponding stationary state is locally stable in accordance with the Eigenvalue criteria (Hinrichsen and Pritchard, 2005): (λ_S = -0.014, λ_P = -23.26, λ_X = -0.022, λ_L = -0.010, λ_A = -0.010, λ_I = -0.010, λ_{Cdl} = -0.0228). The minimum Cd^{2+} removal was at D = 0.1 h^{-1} corresponding to a residence time of 0.41 days (143.1 mg/L with initial concentrations of Cd^{2+} of 170 mg/L). On the other hand, at an initial Cd^{2+} concentration of 300 mg/L at D = 0.01 h^{-1}, metal removal is not complete, possibly because this high Cd^{2+} concentration is significantly toxic for *Desulfovibrio alaskensis* 6SR.

CONCLUSION

Batch cultures of *Desufovibrio alaskensis* 6SR were conducted in media containing different initial Cd^{2+} concentrations. *D. alaskensis* 6SR cells grown in batch process are able to remove nearly all cadmium (99.9%) when the initial concentration was less than 170 mg/L. A proposed mathematical model closely fitted the experimental data in particular at Cd^{2+} concentration under 170 mg/L. The proposed mathematical model accurately predicts the behavior of cadmium removal in a continuous operating system. In this case, an optimum dilution rate (D = 0.01h^{-1}) was found for maximum removal of Cd^{2+} (99.1% at 170 mg/L). The simulation was able to predict the inhibitory effect of cadmium at high concentrations.

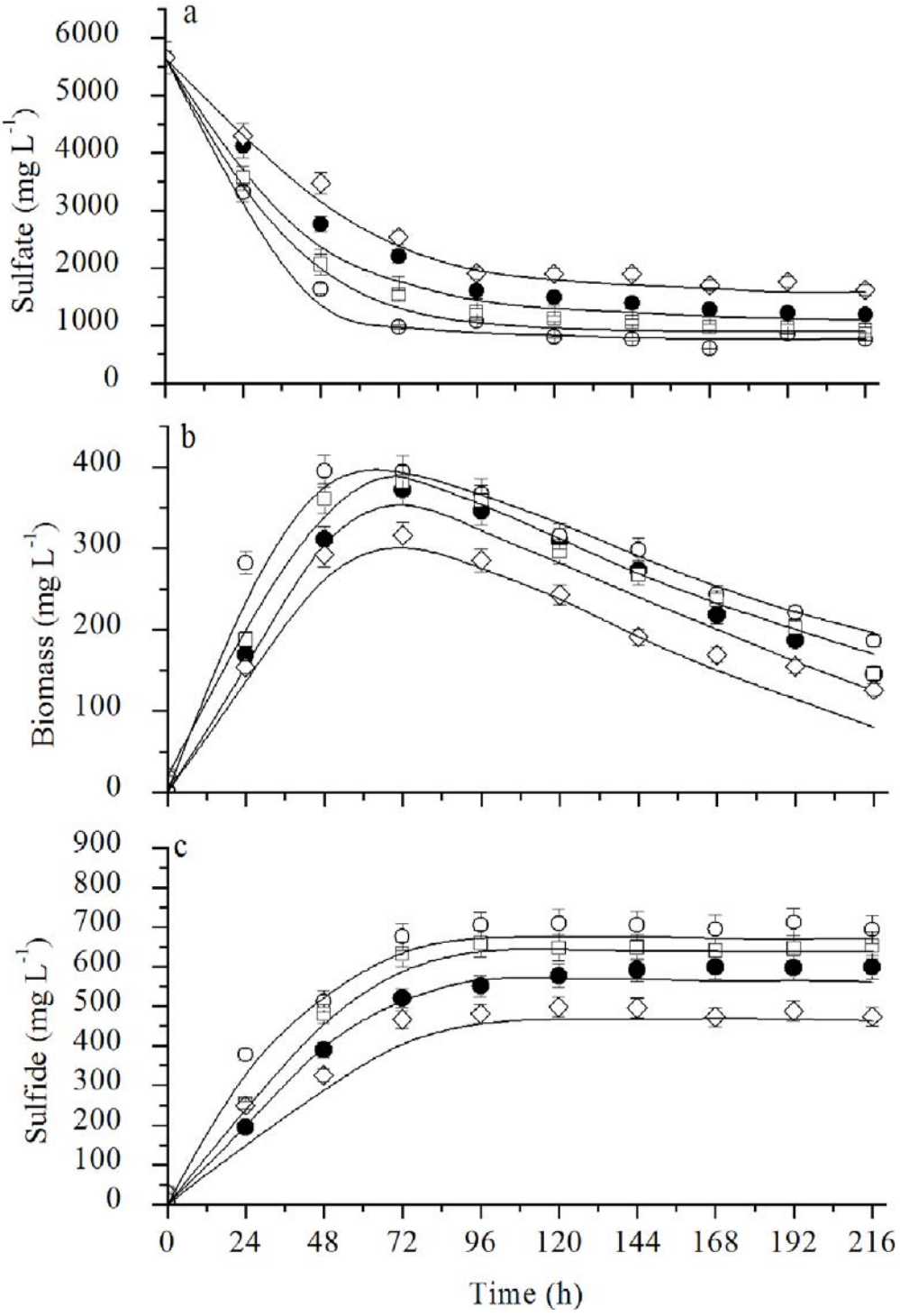

Fig. 2. Comparison of: a) experimental sulfate, b) experimental biomass, and c) experimental sulfide with predictions of the model (——). Batch fermentations with different initial Cd^{2+} concentrations, (○) 50 mg L^{-1}, (□) 100 mg L^{-1}, (●) 170 mg L^{-1}, and (◇) 190 mg L^{-1}

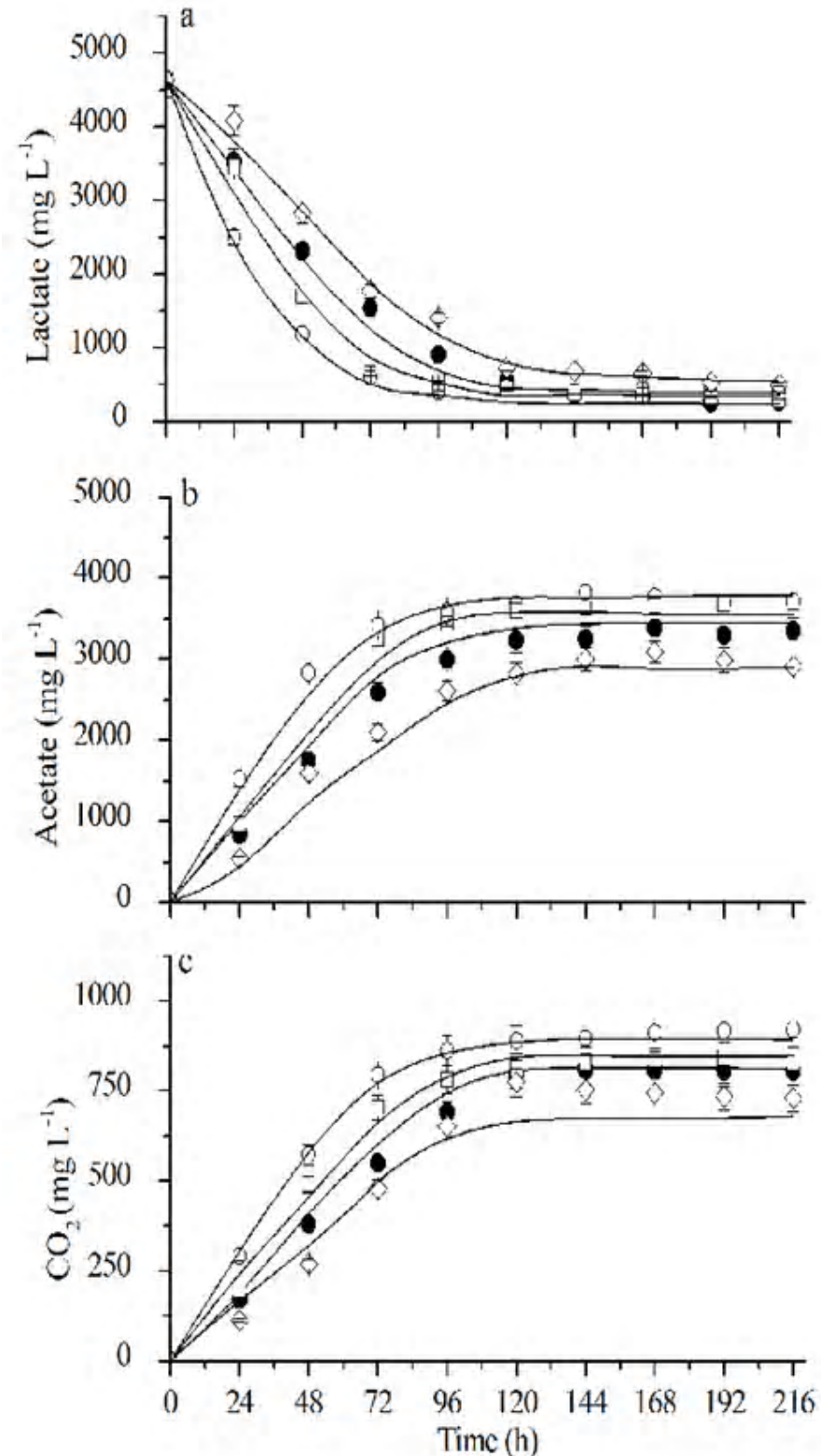

Fig. 3. Comparison of: a) experimental lactate, b) experimental acetate, and c) experimental carbon dioxide with predictions of the model (——). Batch fermentations with different initial Cd^{2+} concentrations, (o) 50 mg L^{-1}, (□) 100m g L^{-1}, (●) 170 mg $L^{-1}$and (◇) 190 mg L^{-1}.

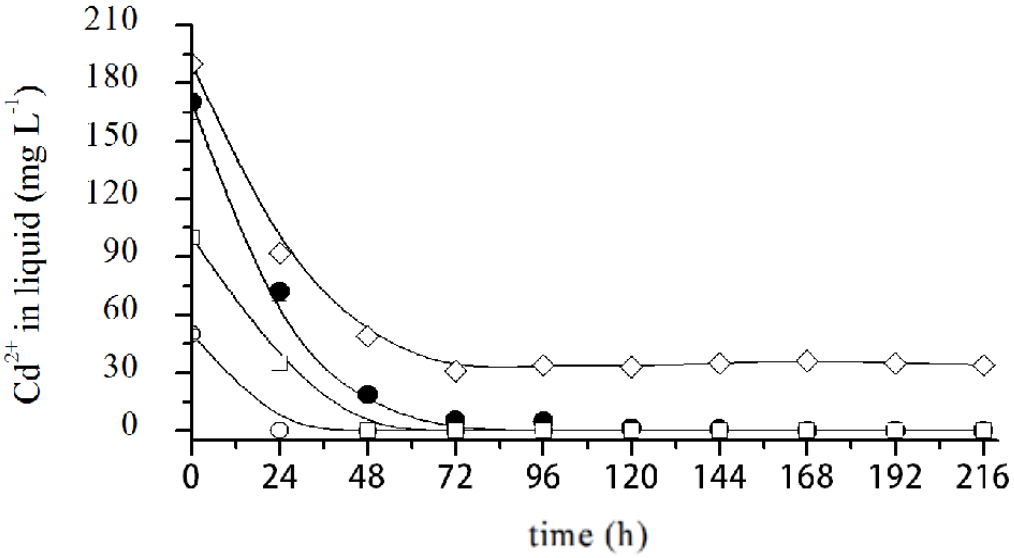

Fig. 4. Comparison of: a) experimental Cd^{2+} in liquid with predictions of the model (——). Batch fermentation with different initial Cd^{2+} concentrations, (o) 50 mg L^{-1}, (□) 100 mg L^{-1}, (●) 170 mg L^{-1}, and (◇) 190 mg L^{-1}.

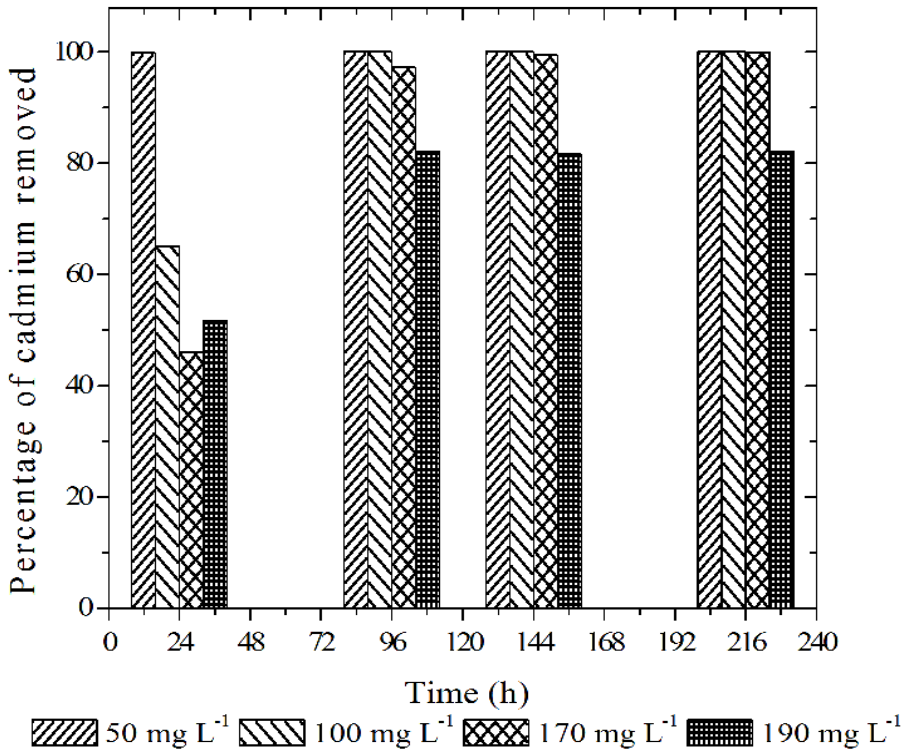

Fig. 5. Percentage of Cd^{2+} removal by *Desulfovibrio alaskensis* 6SR at different initial Cd^{2+} concentrations, 50 mg L^{-1}, 100 mg L^{-1}, 170 mg L^{-1}, and 190 mg L^{-1}

Table 2. Kinetic parameters for the mathematical model (initial cadmium 170 mg·L^{-1})

Parameter	Value	Description	Units
α	1.9 ±0.35	Exponential term for Luong model	Dimensionless
γ	2.38 ±0.55	Exponential term for Cadmium concentration	Dimensionless
δ	2.85±0.3	Exponential term for Moser model	Dimensionless
ε	2.2±0.45	Exponential term for Lactate concentration	Dimensionless
η	0.2±0.05	Exponential term for Moser model	Dimensionless
Y	0.13±0.25	Yield coefficient	Dimensionless
k_{spx}	0.13±0.05	Maximum specific rate	h^{-1}
Y_p	0.9±0.01	Product yield coefficient	Dimensionless
k_p	680±100	Inhibition constant for hydrogen sulfide	mg L^{-1}
k_s	0.0009048± 0.001	Monod saturation constant for organic substrate	mg L^{-1}
k_d	0.008±0.002	Mortality constant	h^{-1}
k_{Cd}	1.9±0.80	Inhibition constant for Cadmium	mg L^{-1}
$k_{umaxCdl}$	0.0002±0.00001	Maximum specific rate for Cadmium	L $h^{-1}mg^{-1}$
k_1	3±1.5	Saturation coefficient	mg L^{-1}
k_2	59±2.3	Inhibition constant for Haldane	mg L^{-1}
$Y_{L/X}$	0.57±0.23	Yield coefficient: Lactate/biomass	Dimensionless
$Y_{A/X}$	0.76±0.13	Yield coefficient: Acetate/biomass	Dimensionless
k_{LA}	2.805±0.5	Maximum specific rate for Lactate/Acetate	h^{-1}
$k_{aœ}$	500±50	Saturation coefficient (Acetate)	mg L^{-1}
k_{lac}	1.85E4±500	Saturation coefficient (Lactate)	mg L^{-1}
k_I	900±50	Saturation coefficient (carbon dioxide)	mg L^{-1}
$Y_{I/X}$	1± 0.3	Yield coefficient: carbon dioxide /biomass	Dimensionless
k_{co}	0.12±0.050	Maximum specific rate for carbon dioxide	h^{-1}

Table 3. Model efficiency and correlation coefficients for the mathematical model

	R^2	Model efficiency (Ω)
Hydrogen sulfide	0.9939	0.9957
Sulfate	0.9796	0.9927
Biomass	0.9966	0.9733
Liquid Cd^{2+}	0.9956	0.9980
Lactate	0.9984	0.9956
Acetate	0.9984	0.9976
Carbon dioxide	0.9984	0.9975
Global	0.9940	0.9929

Table 4. Comparison of kinetic parameters reported for SRB

SRB	k_s (mgL^{-1})	k_p (mgL^{-1})	k_d (h^{-1})	μ_{max} (h^{-1})	Y	M$^+$	Type of operation	References
Desulfovibrio alaskensis 6SR	**0.009**	**680**	**0.008**	**0.13**	**0.13**	●	**Batch**	**In this work**
A mixed culture SRB	0.94 to 0.01	—	0.035	0.06	0.15	o	Continuous	Moosa *et al.*, (2002)
A mixed culture SRB	0.02 to 0.12	—	0.035	0.06	0.56 to 0.60	o	Continuous	Moosa *et al.*, (2005)
SRB	0.13	—	—	1.08		o	Batch	Nielsen *et al.*, (1987)
SRB	0.001	550	—	0.12		o	Batch	Kalyuzhnyi *et al.*, (1998)
Desulfovibrio alaskensis	—	—	—	0.13		o	Batch	Feio *et al.*, (2004)
Desulfonema magnum	120	—	—	0.007	0.0965	o	Batch	O´Flaherty *et al.*, (1998)
Desulfotomacolum acetoxidans	100	—	—	0.02	0.11	o	Batch	O´Flaherty *et al.*, (1998)
Consortium	—	—	—	0.02 to 0.05	0.02	o	Batch	Dinkel *et al.*, (2010)
Desulfovibrio desulfuricans G20	—	—	—	0.14	—	●	Shaker operated	Sani *et al.*, (2001)
Desulfovibrio desulfuricans G20	—	—	—	0.13	—	●	Shaker operated	Sani *et al.*, (2001)
Desulfovibrio desulfuricans G20	—	—	—	1.86	—	●	Shaker operated	Sani *et al.*, (2001)
SRB	142.7	—	—	3.71	—	●	Batch	Kaksonen *et al.*, (2004)
SRB	100	200	—	0.02	0.01	●	Batch	Al-Zuhair *et al.*, (2008)
Desulfovibrio, Desulfotomaculum	2.5-3.7	356	—	0.19-0.22	—	●	Batch	Oyekola *et al.*, (2010)

o SRB without metal effect ● SRB with metal effect — No data

k_d: mortality constant , k_s: saturation constant, k_p: inhibition constant and Y: yield coefficient for cell growth

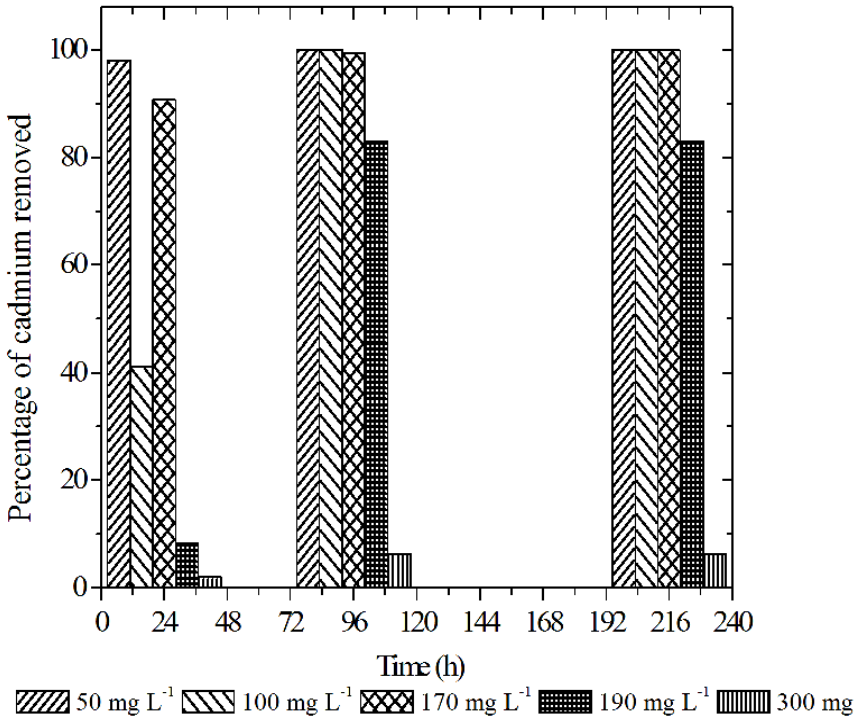

Fig. 6. Model prediction for percentage of Cd^{2+} removal at different initial Cd^{2+} concentrations in a continuous process, D= *0.01* h^{-1}.

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