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A Study on the Derivation of a Mean Velocity Formula from Chiu's Velocity Formula and Bottom Shear Stress

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ABSTRACT: This study proposed a new discharge estimation method using a mean velocity formula derived from Chiu's 2D velocity formula of probabilistic entropy concept and the river bed shear stress of channel. In particular, we could calculate the mean velocity, which is hardly measurable in flooding natural rivers, in consideration of several factors reflecting basic hydraulic characteristics such as river bed slope, wetted perimeter, width, and water level that are easily obtainable from rivers. In order to test the proposed method, we used highly reliable flow rate data measured in the field and published in SCI theses, estimated entropy M from the results of the mean velocity formula and, at the same time, calculated the maximum velocity. In particular, we obtained $\phi(M)$ expressing the overall equilibrium state of river through regression analysis between the maximum velocity and the mean velocity, and estimated the flow rate from the newly proposed mean velocity formula. The relation between estimated and measured discharge was analyzed through the discrepancy ratio, and the result showed that the estimate value was quite close to the measured data.

Key words: Hydraulic characteristics, Mean velocity formula, F(M) Equation, Entropy parameter M, Equilibrium state $\phi(M)$

INTRODUCTION

Korea has rainfall with large seasonal variation, and because of its east-high and west-low landform, the river slope is steep and the length of main channel is short. For these reasons, flood is discharged at once and therefore, the country is very vulnerable to water-related disasters. In order to overcome the natural environment and to achieve the national status as a world power in water control, the Korean government is promoting the Four Major Rivers Restoration Project, investing 15.4 trillion won from June 2009 as a part of the "Green New Deal" policies. This is a very important national project, and the largest river design and construction work in Korean history. Despite its scale, however, the project aims to complete weirs, reservoirs, and various linked projects within a relatively short period until 2012. Thus, it is quite important to estimate accurately how the project will change river environment. The dredging of rivers will change the river bed as well as the courses in the longitudinal and transverse directions, and the managed stage of weirs and change in water level by sluice gate operation are expected to influence the level of river water as well as nearby groundwater. What is more, water quality and ecosystem will be affected, and the characteristics of tributaries to the main rivers will also suffer radical changes. Thus, it is crucial to protect human lives and properties from disasters that may be caused by such radical changes of river environment.

Accordingly, the fast and accurate estimation of discharge is a prerequisite for preventing and coping with disasters. In order to estimate highly reliable discharge, which is an important element in planning, evaluating and managing water resources and in designing hydraulic structures, it is essential to develop a mean velocity formula that reflects the hydraulic characteristics of the river.

Previous studies on discharge estimation in Korea and other countries are as follows. Leon, *et al.*(2006) analyzed the relation between stage and discharge and proposed a discharge estimation method using the Muskingum Cunge (M.C) model based on highly quantitative spatial data in the Negro River on the Amazon Basin. Sahoo and Ray (2006) analyzed the correlation between stage and discharge for the Hawaii

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Basin by applying Artificial Neural Network (ANN), and developed a model for estimating the discharge of natural rivers. In addition, Oh *et al.* (2005) estimated the mean velocity of entire cross-section and then estimated discharge using linear continuity equation in order to improve the conventional estimation of flood discharge based on the stage- discharge curve for most of Korean rivers. Moreover, Lee *et al.* (2009) analyzed the results of field measuring using an electronic float system developed with GPS and RF communication, and proposed a discharge measuring method. On the other hand, Choo (2002) implemented velocity distribution using point velocity in Chiu's 2D velocity distribution formula, and proposed a river discharge estimation method by applying the velocity distribution to Chiu's 2D mean velocity formula. What is more, Kim *et al.*(2008) proposed a flow rate estimation method for natural rivers using Chiu's velocity distribution formula and maximum velocity estimation. Choo *et al.* (2011) developed newly discharge estimation by using the Manning and Chezy equation method reflecting hydraulic characteristics.

However, these previous studies showed limitations in reflecting the hydraulic characteristics of each river and were somewhat unsatisfactory in terms of reliability.

This study proposed a formula for estimating the mean velocity of river using factors easily obtainable from rivers including the unique hydraulic characteristics of a river such as area, width, wetted perimeter and river bed slope. The formula was derived from Chiu's 2D velocity formula of probabilistic entropy concept and the river bed shear stress of channel.

MATERIALS& METHODS

Chiu's velocity formula, which is also known as Natural Raw, is a 2D velocity formula that expresses well the distribution of velocity from the bottom to the surface of a channel. This formula applies hydraulically the concept of entropy maximization used in probability and statistics, and the detailed derivation process is available in Chiu(1978), Chiu(1987), Chiu(1988), Chiu and Chen(2002), Chiu and Tung(2002), Choo(2002). Accordingly, the entropy function of velocity can be expressed as follows.

$$H = -\int_0^{u_{max}} f(u) \ln f(u) du \tag{1}$$

u is the time mean velocity distributed spatially over the cross-section of channel, u_{max} indicates the maximum velocity, and $f(u)$ is the probability density function of velocity. $f(u)$ includes equation (2) and (3) as effective information imposing constraints for the maximization of entropy.

$$\int_0^{u_{max}} f(u) du = 1 \tag{2}$$

$$\int_0^{u_{max}} u f(u) du = \bar{u} = Q / A \tag{3}$$

Equation (2) is the definition or condition of $f(u)$ as a probability density function. Equation (3) should be derived in a way that $\bar{u}$ satisfies effective information on it. For example, $\bar{u}$ should be equal to Q / A, or to what is given to an experimental formula like Manning's equation given by Manning's n, hydraulic radius and the slope or energy grade line of channel. From the application of the Method of Lagrange maximizing H to the limiting factors of equation (2) and (3), we can derive equation (4) as follows.

$$f(u) = e^{\lambda_1 - 1} e^{\lambda_2 u} = e^{a_1} e^{a_2 u} (0 \le u \le u_{max}) \tag{4}$$

For a more detailed derivation process using additional limiting factors and other possible results, see Chiu(1987), Chiu(1989).

As in Fig. 1, Chiu's velocity distribution equation uses $\xi - \eta$ coordinate system consisting of isovels ξ and η that connect points with the same velocity on the cross-section. A one-to-one relation is established between position on the cross-section expressed as isovel and velocity, and the entropy velocity distribution equation is derived using the cumulative probability function for velocity u on isovel ξ.

Accordingly, in the 2D cross-section coordinate system, velocity for a specific point is defined as equation (5).

$$\int_0^{u} e^{a_1 + a_2 u} du = \frac{\xi - \xi_0}{\xi_{max} - \xi_0} \tag{5}$$

If equation (4) is substituted into equation (2) and rearranged, and then $a_2 u_{max}$ is replaced with M, equation (6) is obtained. Here, M is a parameter indicating velocity distribution.

$$\frac{e^{a_1}}{a_2} = (e^M - 1) \tag{6}$$

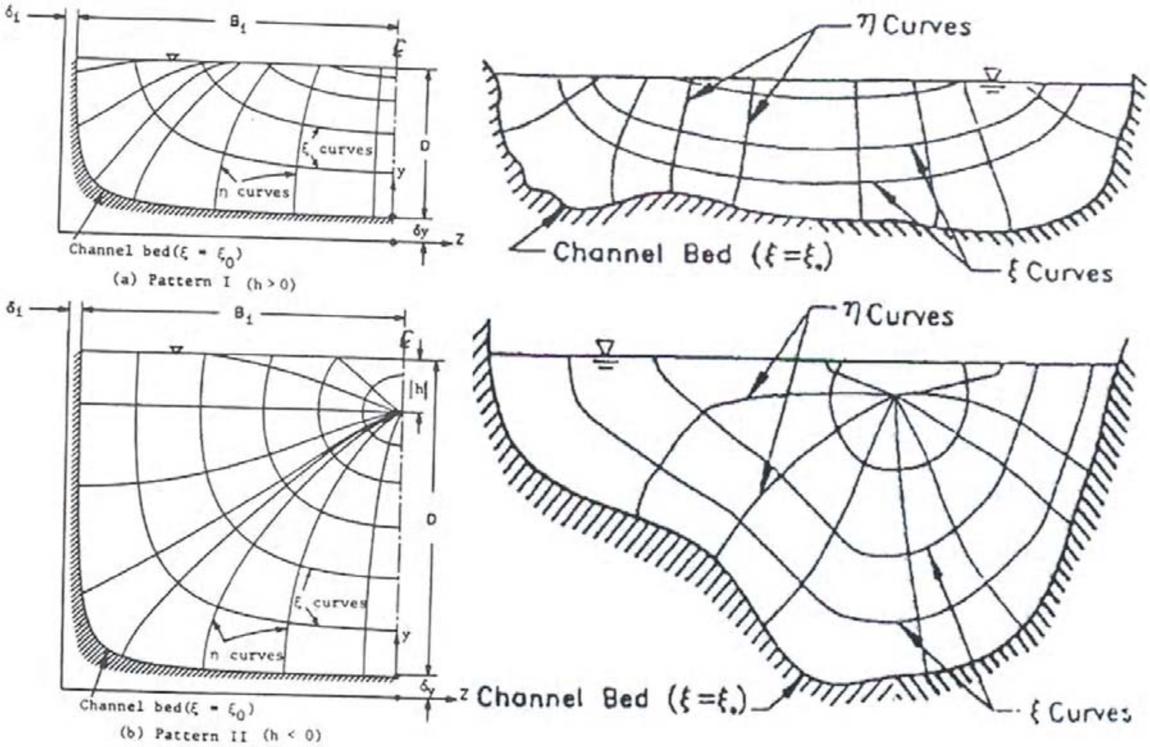

Fig.1. Coordinates in open-channel sections(chiu,1988,1989)

If equation (5) is solved and rearranged using equation (6), a 2D velocity distribution formula is obtained as in equation (7).

$$u = \frac{u_{max}}{M} ln[1+(e^M - 1)\frac{\xi - \xi_0}{\xi_{max} - \xi_0}] \qquad (7)$$

In addition, if equation (4) is substituted into equation (3) and is solved, a 2D mean velocity equation is obtained as in equation (8).

$$\frac{\bar{u}}{u_{max}} = \frac{e^M}{e^M - 1} - \frac{1}{M} = \phi(M) \qquad (8)$$

This equation can be simplified to equation (9).

$$\bar{u} = \phi(M) u_{max} \qquad (9)$$

Where, $\phi(M)$ is an indicator showing the linear relation between the mean velocity and the maximum velocity as in equation (8), and is called equilibrium state $\phi(M)$.

If equation (4) is substituted into equation (2) and (3) and rearranged, equation (10) is obtained.

$$u_{max} e^{a_1} = \frac{M}{(e^M - 1)} \qquad (10)$$

If equation (8) is substituted into equation (10) and rearranged, equation (11) is obtained.

$$\bar{u} e^{a_1} = (\frac{M}{e^M - 1})(\frac{e^M}{e^M - 1} - \frac{1}{M}) = (e^M - 1)^{-1}[Me^M(e^M - 1)^{-1} - 1] \qquad (11)$$

On the other hand, if the bottom shear stress of channel is expressed as equation (12) and $\frac{du}{d\xi}$ is estimated from equation (5) and rearranged, the results are as in equation (13) and (14).

$$\tau_0 = \mu[\frac{du}{dy}]_{y=y_0} = \mu[\frac{1}{h_\xi}][\frac{du}{d\xi}]_{\xi=\xi_0} \qquad (12)$$

Where, τ_0 is bottom shear stress, μ the viscosity coefficient of fluid, and h_ξ unit conversion factor indicating length unit dy by multiplying by $d\xi$.

In addition, the mean shear stress can be expressed as equation (13).

$$\overline{\tau_0} = \mu [\frac{1}{\overline{h_\xi}}][\frac{du}{d\xi}]_{\xi=\xi_0} = \rho g R I_f \quad (13)$$

Where, $\overline{\tau_0}$ is the mean shear stress of the bottom boundary layer, $\overline{h_\xi}$ the mean value of h_ξ according to the channel boundary layer, ρ water density, g gravity acceleration, R hydraulic radius, and I_f energy gradient.

$\frac{du}{d\xi}$ in equation (13) can be expressed as equation (14) from equation (5).

$$\frac{du}{d\xi} = [\frac{1}{(\xi_{\max} - \xi_0) f(u)}] = [\frac{1}{(\xi_{\max} - \xi_0) e^{a_1 + a_2 u}}] \quad (14)$$

Because $u = 0$ in the bottom boundary layer of channel, $\xi_0 = 0$ and $\xi_{max} = 1$ and, as a result, $\xi_{max} - \xi_0 = 1$. Accordingly, equation (14) is rearranged to equation (15).

$$[\frac{du}{d\xi}]_{\xi=\xi_0} = [\frac{1}{e^{a_1}}] \quad (15)$$

If equation (15) is substituted into equation (13) and rearranged, equation (16) is obtained.

$$e^{a_1} = \frac{\mu}{\overline{h_\xi} \rho g R S_f} \quad (16)$$

If equation (16) is substituted into equation (11) and rearranged, new mean velocity is derived as in equation (17).

$$\bar{u} = \frac{\overline{h_\xi} g R S_f}{\nu F(M)} \quad (17)$$

Where, $$F(M) = (e^M - 1)[Me^M (e^M - 1)^{-1} - 1]^{-1} \quad (18)$$

Accordingly, equation (17) means that if there are measured values of $F(M)$, $\overline{h_{\xi_0}}$, g, R, S_f, ν indicating the hydraulic characteristics of river, we can calculate the mean velocity and estimate the flow rate by multiplying the velocity by cross-sectional area. Newly proposed flow rate estimation method based on the mean velocity formula is here.

Based on Chiu's 2D velocity formula using the probabilistic entropy concept, a mean velocity formula was derived as in equation (17) from the relation between the sum of kinematic coefficient of viscosity and velocity gradient perpendicular to the channel boundary and the mean shear stress formula. This equation has as its terms the hydraulic characteristic factors easily measurable from rivers. $F(M)$ is estimated from equation (17) by substituting the measured values of mean velocity, river bed slope, hydraulic radius, kinematic coefficient of viscosity, etc., and then entropy parameter M is calculated. Using the calculated M, $\phi(M)$ is calculated from equation (8). And u_{max} is also calculated by the equation (8). With all data, $\phi(M)$ at the equilibrium of the whole river was calculated. From the theoretical relation between the measured mean velocity and each , we can calculate maximum velocity , which is hardly measurable accurately in open channels. Through this process, the mean velocity was estimated using the relation between maximum velocity and overall equilibrium state . The detailed process was summarized as in Table 1.

Overall equilibrium state $\phi(M)$ by the relation between the mean velocity and the maximum velocity is as follows.
In order to verify the contents of this study, we used data on 4 laboratory channels measured by Abdel-Aal(1969), Bengal(1965), Chyn(1935), and Costello (1974), and field data measured in 3 natural rivers including Mahmood *et al.*(1979),Shinohara and Tsubaki(1959), and Leopold(1969) among highly reliable flow rate survey data provided by Peterson and Howells(1973).Table 2 and Fig. 2, 3 show the results of estimating overall equilibrium state $\phi(M)$ that means the gradient in the linear relation between estimated maximum velocity u_{max} and measured mean velocity u_{mean} and determination coefficient R^2 that indicates the accuracy of the value.

RESULT& DISCUSSION

Table 2 and Fig. 2, 3 show clearly the values of overall equilibrium state $\phi(M)$, which is the gradient

Table 1. The Procedure for estimation method of discharge by F(M) equation

Estimate M by substituting $\bar{u}$, $\overline{h_{\xi_0}}$, g, R, S_f, ν for each cross-section of channel into Equation (18), and then estimate $\phi(M)$
↓
Estimate the maximum velocity of each cross-section from Equation (9) using measured mean velocity and calculated $\phi(M)$
↓
From Equation (8), estimate overall equilibrium state $\phi(M)$, which means the gradient of the linear relation between calculate u_{max} and measured $\bar{u}$ and reflects the general characteristics of the river
↓
Estimate $\bar{u}$ of each cross-section newly using u_{max} estimated for each cross-section and overall equilibrium state $\phi(M)$
↓
Test the accuracy of the flow rate based on estimated $\bar{u}$ and measured flow rate using the discrepancy ratio

Table 2. Linear regression analysis between u_{mean} and u_{max}

Data		$u_{mean} = \phi(M) u_{max} (y = ax)$	$\phi(M)$	R^2
Lab	Abdel-Aal, F.M(1969)	$u_{mean} = 0.8657 u_{max}$	0.8657	0.9998
	Govt. of W. Bengal(1965)	$u_{mean} = 0.8197 u_{max}$	0.8197	0.9987
	Chyn, S.D(1935)	$u_{mean} = 0.8521 u_{max}$	0.8521	0.9998
	Costello, W.R.(1974)	$u_{mean} = 0.8667 u_{max}$	0.8667	0.9998
River	Acop Canaldata of Mahmood, et al.(1979)	$u_{mean} = 0.9110 u_{max}$	0.9110	0.9999
	Hii River data of Shinohara, K. and Tsubaki, T.(1959)	$u_{mean} = 0.8824 u_{max}$	0.8824	0.9992
	Leo-River data of Leopold, L.B.(1969)	$u_{mean} = 0.9146 u_{max}$	0.9146	0.9999

in the linear relation between the mean velocity and the maximum velocity estimated through the process in Table 1 using data measured in laboratory channels and natural rivers. The mean velocity of river was estimated using equation (17) and the flow rate was estimated by multiplying the estimated mean velocity by the cross-sectional area of each laboratory channel or river. Standard deviations, which indicate the variation of mean value between measured discharge data and estimate flow rate data for the laboratory channels and natural rivers, were 0.00006, 0.00009, 0.00016 and 0.00033, and 0.1151, 0.0144 and 1.1791, respectively. A small standard deviation means that measured discharge is almost equal to estimated discharge. The reason that standard deviation is larger in natural rivers than in laboratory channels is probably that the discharge of natural rivers is relatively higher than that of laboratory channels. Detailed results are as in Fig. 4 and Fig. 5.

As presented above, the proposed mean velocity formula in equation (17) estimated easily the maximum velocity that enables us to cope with the fluctuation of stage, and the accuracy was also high. This means that even during the flood season when the stage is high we can obtain the mean velocity of cross-sections easily from the maximum velocity.

In order to verify the results above, we analyzed the results using the discrepancy ratio, which is the common logarithm of the ratio between measured and estimated discharge as in Fig. 6, 7. A discrepancy ratio larger than 0 means overestimation, and that smaller than 0, namely, a negative value means underestimation. Through this, we compared the distribution of discharge estimated by equation (17) the newly proposed mean velocity formula with the distribution of measured discharge. The error range was between -0.02 and 0.03 for laboratory channels and between -0.015 and 0.02 for natural rivers, proving that the two values were almost equal to each other.

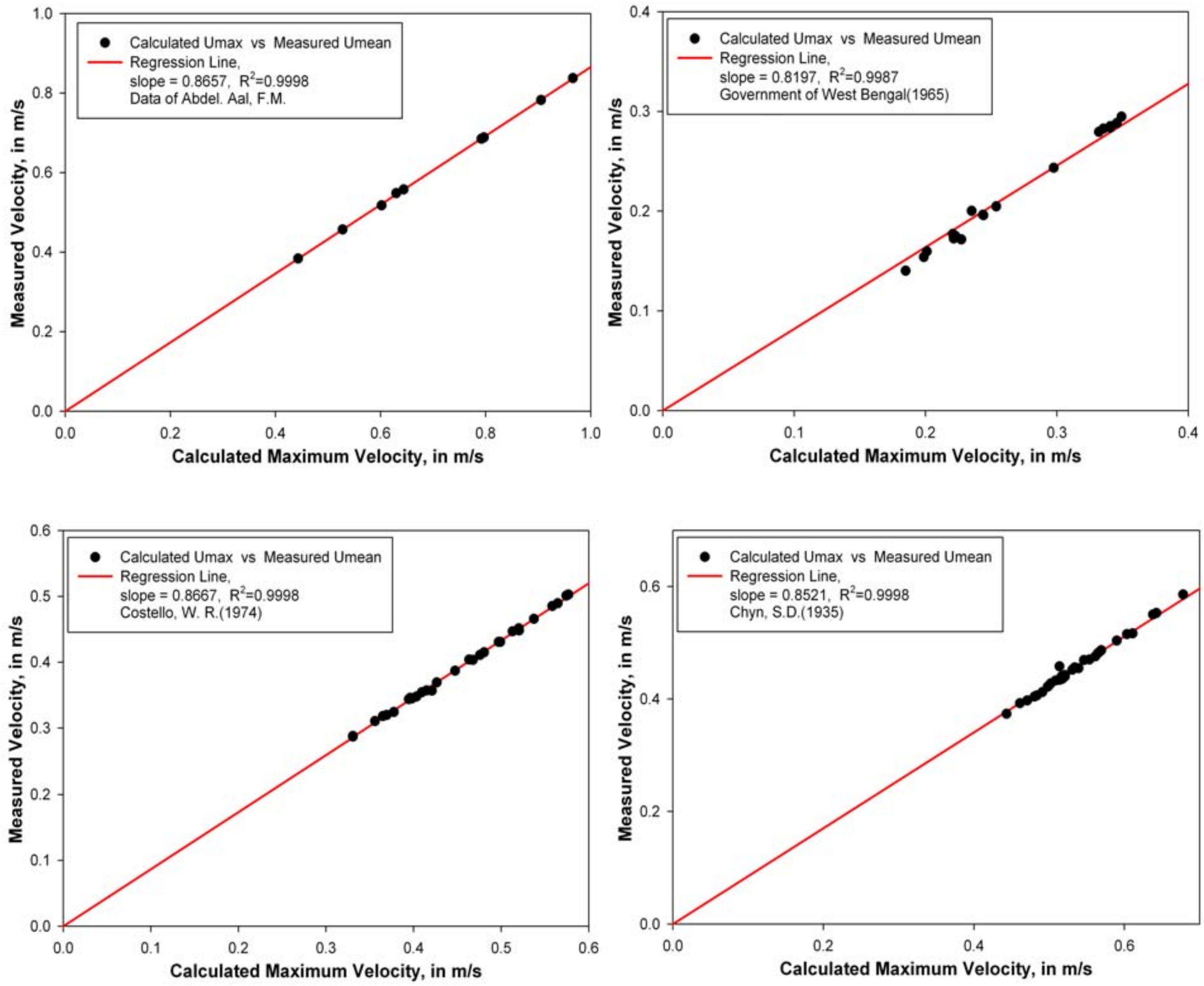

Fig. 2. The Relationship between Measured Mean velocity and Caculated Maximum Velocity in the lab

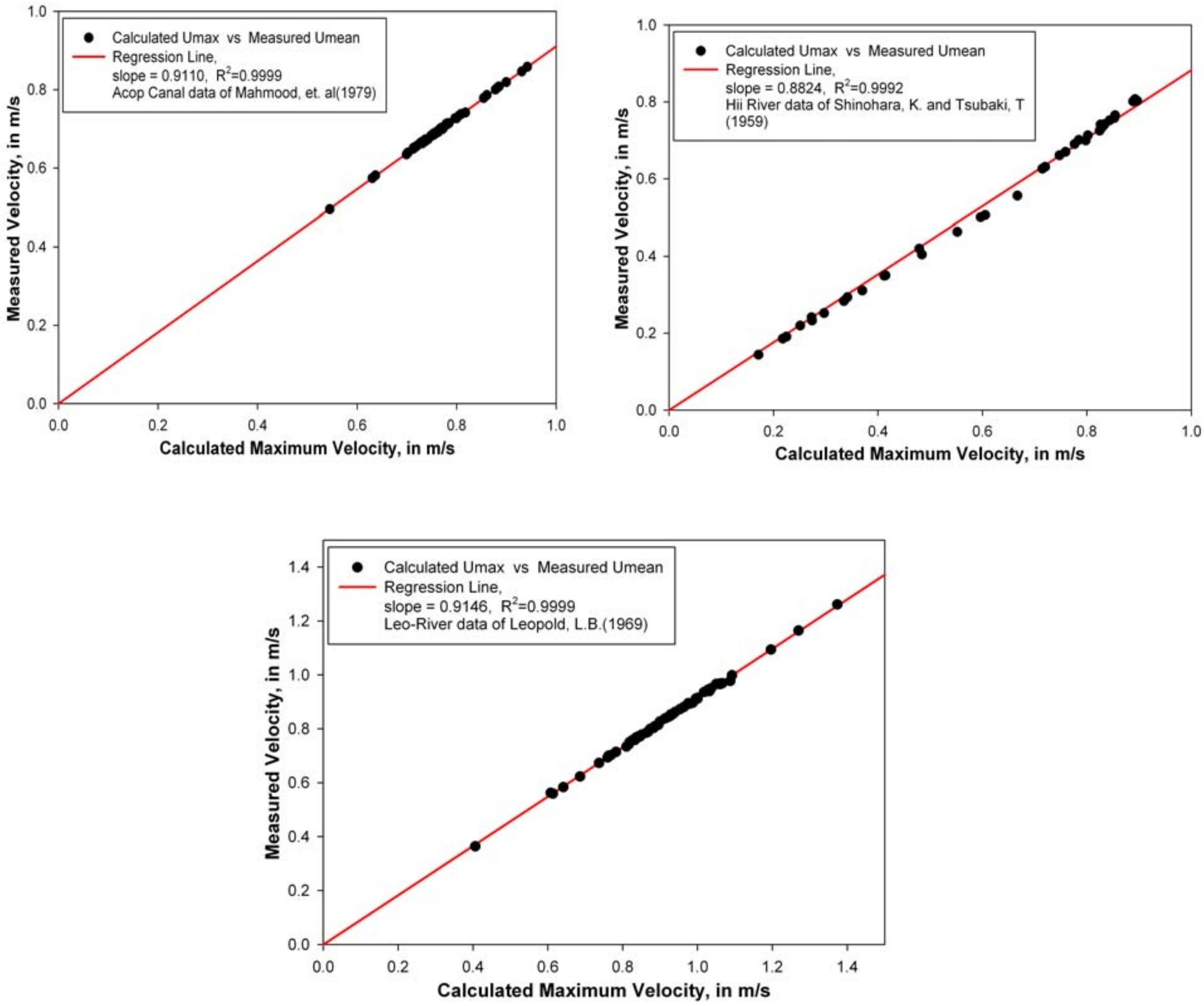

Fig. 3. The Relationship between Measured Mean velocity and Caculated Maximum Velocity in the natural open

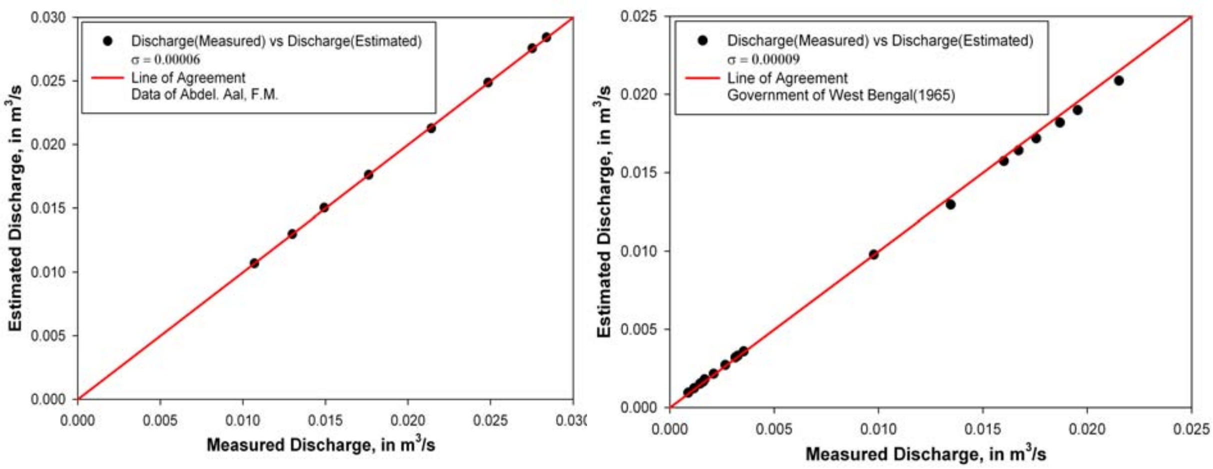

Fig. 4. Analysis Results by using proposed mean velocity Eq. in the lab(Continues)

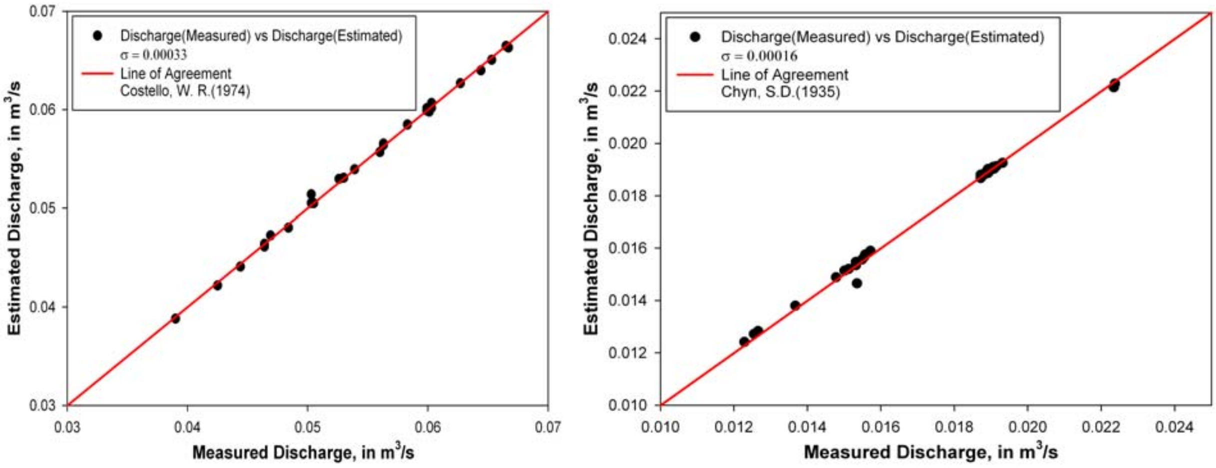

Fig. 4. Analysis Results by using proposed mean velocity Eq. in the lab

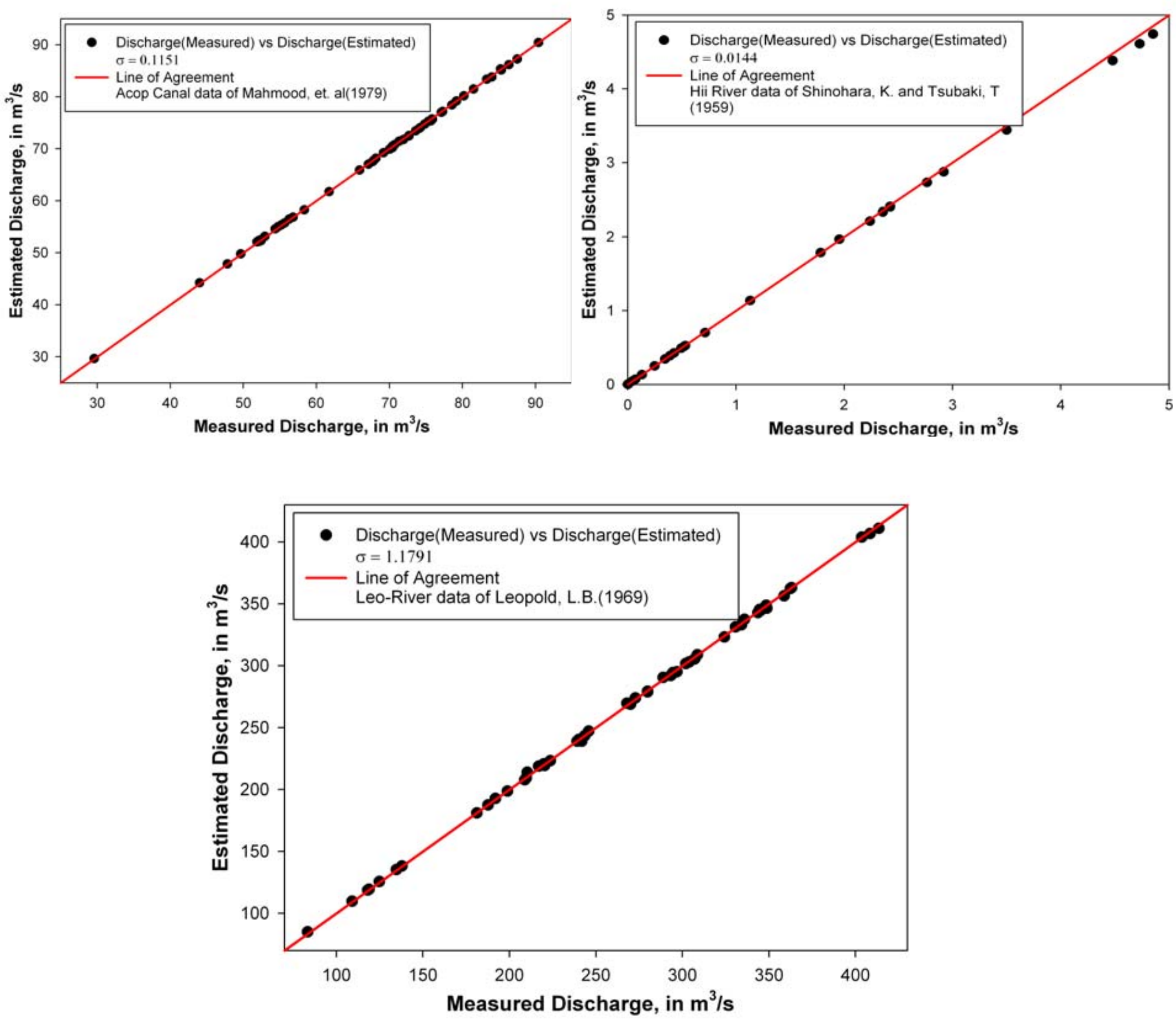

Fig. 5. Analysis Results by using proposed mean velocity Eq. in the natural open

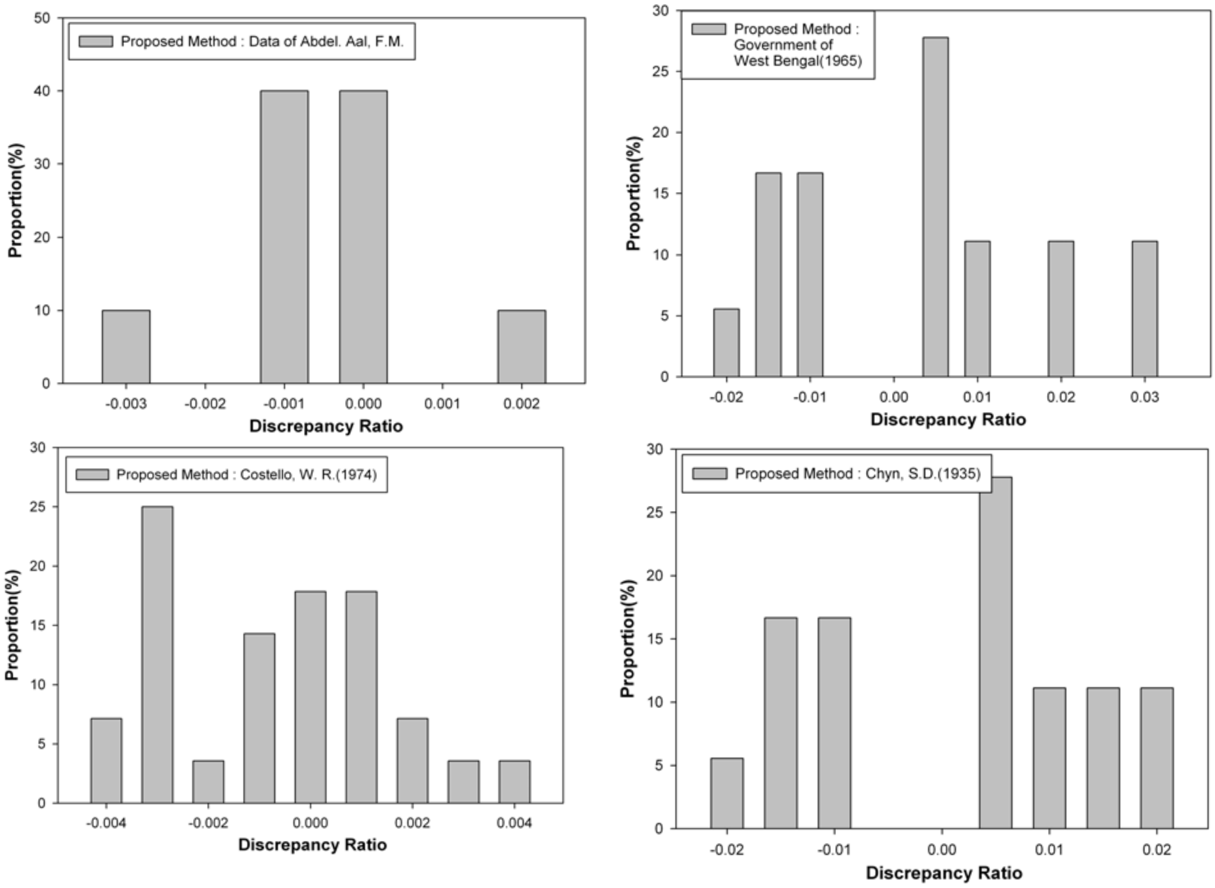

Fig. 6. Discrepancy Ratio Analysis Between Measured and Estimated Discharge in the Lab

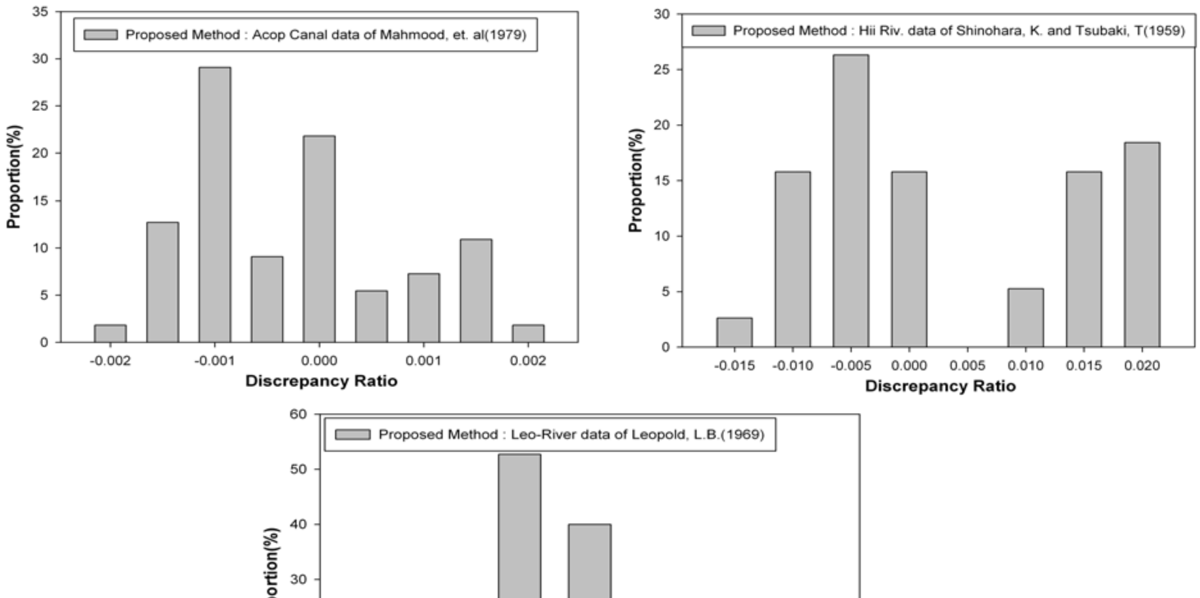

Fig. 7. Discrepancy Ratio Analysis Between Measured and Estimated Discharge in the natural open

CONCLUSION

This study developed a mean velocity formula derived from Chiu's 2D velocity formula using the probabilistic entropy concept and the river bed shear stress of channel. In particular, the developed new velocity formula reflects accurately hydraulic characteristics such as water level, width, hydraulic radius and river slope easily obtainable from rivers, and can estimate accurately the maximum velocity that is hardly measurable in natural rivers.For this study, we used reliable data measured from laboratory channels and natural rivers. According to the results, standard deviations for the laboratory channels were 0.00006, 0.00009, 0.00016, and 0.00033, respectively, and those for the natural rivers were 0.1151, 0.0144, and 1.1791, showing that estimated data are quite close to measured data.In order to verify the result, we analyzed using the discrepancy ratio, and the error range was between -0.02 and 0.03 for laboratory channels and between -0.015 and 0.02 for natural rivers, proving that the two values were almost equal to each other.The mean velocity formula calculated from equation (18) showing very high accuracy is expected to make a great contribution to the accurate estimation of discharge, which is most important for water control in river environment that may change radically after the Four Major Rivers Restoration Project. Furthermore, if this formula is refined further through continuous research, we may be able to estimate flow rate relatively accurately even during the dry season and the flood season, in which field measuring has been quite difficult, and to use the formula as a theoretical tool for real-time discharge measuring systems.

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