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A Partial Linearization Method for Multi-Objective Continuous Network Design Problem with Environmental Considerations

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ABSTRACT: Nowadays, the environmental impact of transportation project and, especially air pollution impacts, are major concerns in evaluating transportation projects. Based on this concern, beside definition of traditional objective functions like total travel time and total investment cost, different type of environmental related function is considered as objective function in urban network design. In this paper, the continuous network design problem (CNDP) is formulated as a multi-objective bi-level optimization problem. The objective function of the upper level problem is a weighted combination of total travel time, total investment cost and total vehicular emission in the network. The lower level problem is the traffic assignment problem which would predict the vehicular flow on each link in the network. A new solution algorithm is proposed for solving the bi-level optimization problem by the partial linearization of the lower level problem. The solution algorithm was applied to the city of Sioux Falls, a well known transportation network.

Key words: Environmental impact, Transportation network design, Bi-level optimization, Partial linearization

INTRODUCTION

Design Problem (NDP) is the problem of selecting optimal capacity enhancement in transportation networks. NDP is usually classified to three types of Continuous Network Design Problem (CNDP) as in Dantzig et al. (1979), Marcotte (1983), Harker and Friesz (1984), Le Blanc and Boyce (1986), Suwansirikul et al. (1987), Jahangiri et al. (2011), and Meng et al. (2001), Discrete Network Design Problem (DNDP) which was used in Billheimer and Gray (1973), Los (1979), Boyce and Janson (1980), Poorzahedy and Turnquist (1982), Herrmann et al. (1996), Solanki et al. (1998), Le Blanc (1975) and Gao et al. (2005), Afandizadeh et al. (2006), and Mix Network Design Problem (MNDP) as in Afandizadeh et al. (2011). In the CNDP, the decision variables, capacity enhancement, are continuous variables; but, in the DNDP, the decision variables are integers - mostly the problem is to decide whether to implement a network improvement project or not. The MNDP is a problem in which the capacity enhancement decisions are both integer and continuous variables. For a comprehensive review, the reader may refer to Magnanti and Wong (1984) and Yang and Bell (1998) as examples.The NDP is traditionally formulated as a bi-level optimization problem (Yang and Bell (1998)). In previous studies, the upper-level problem is to minimize the total system's travel time while the lower level problem minimizes the individual drivers' travel time by the equilibrium traffic assignment problem. The network design problem could be considered as a Stackelberg game (Murphy et al. (1982)) in which the upper level problem is the leader problem, whose objective is to minimize the total system cost. The lower level problem is the follower problem, which based on the leaders decision, tries to selfishly minimize his/her individual travel time based on the leaders' decision.

Different objectives have been used as the upper level problem in the past studies (Yang and Bell (1998)). The objective functions such as total travel time, total system cost and social welfare are some of the most common objective functions in the literature. However, none of these researches have taken into account the environmental impact of these network decisions.

Former investigations have shown a wide spectrum of solution algorithms for solving the NDP.

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Dantzig et al. (1979) used the system optimal flow instead of user equilibrium flow in order to expedite the solution of NDP. Neglecting the congestion effect on travel time was another method used by Boyce et al. (1973) and Holmberg and Hellstrand (1998). Expediting the solution procedure through the aggregation of the network by link and node abstraction or extraction was another method that was proposed in Haghani and Daskin (1983). Yang and Bell (1998) facilitated the design procedure using an intrinsic approach which defined a new design problem lacking the complexity of NDP. Heuristic and meta-heuristic methods were also used in many previous studies (Poorzahedy and Turnquist, (1982); Chen and Sul Alfa (1991); Yin (2000); Friesz et al. (1992); Lee and Yang, (1994); Cantarella et al. (2002) and Poorzahedy and Abulghasemi (2005)).

In this paper, the bi-level optimization problem is converted into a problem with a linear lower level problem in which its gradient could be easily calculated and thus the sub-gradient of the upper level problem could be determined. Meanwhile, by considering the effect of network decisions on air pollution, a new aspect of transportation network design was taken into account.

This paper is organized as follows: in the second section, the Multi-objective bi-level Continuous Network Design Problem (MCNDP) is formulated. Section three presents the proposed solution algorithm for the MCNDP. In Section four, the presented method is applied to the network in the city of Sioux Falls. Finally, conclusions and future works are presented in Section five.

MATERIALS & METHODS

As mentioned previously, the continuous network design problem could be formulated as an optimization problem. It is assumed that $G(N,A)$ is a directed graph where N is the set of nodes and A is the set of all links in the graph G. The set A could be partitioned into two subsets, namely A_b and A_p, the latter of which is the set of all links in the basic network and the former is the set of candidate links for capacity expansion. The demand set of the network G is defined as the set of triplets (r,s,q^{rs}) so that, r is the origin node and $r \in N$, s is the destination node and $s \in N$, and q^{rs} is the demand between r and s. Then, by defining C as the vector of link capacities and f as the vector of path flows, the network design problem could be formulated as in Eq. (1):

$$\left[C^*, f^*\right] = Arg_{C \in \Delta} \min Z(C, f^*(C)) \quad (1)$$

Subject to:

$$f^* = Arg_{f \in \Omega} \min g(C)$$

Where C^* is the vector of optimal capacities and f^* is the vector of optimal path flow on the network. Δ and Ω are the feasible solution sets for capacity and path flow, respectively. The problem is to find the optimal capacity expansion and traffic flow subjected to the user equilibrium traffic assignment constraint which is, in turn, a function of capacity. As could be seen, this problem is an NP-Hard bi-level optimization problem.

More specifically, the problem could be formulated as in Eq. (2):

$$\min_C Z_T \quad (2)$$

$$\min_C Z_C$$

$$\min_C Z_E$$

Subject to:

$$c_a^b \le c_a \le c_{\max} \qquad \forall a \in A_p$$

$$\min_X \sum_{a \in A} \int_0^{x_a} t_a(c_a^*, w)\,dw$$

Subject to:

$$\sum_{p \in P^{rs}} f_p^{rs} = q^{rs} \qquad \forall rs \in W$$

$$x_a = \sum_{rs \in W} \sum_{p \in P^{rs}} f_p^{rs} * \delta_{a,p}^{rs}$$

In the above equation:

c_a = the capacity of link $a \in A_p$,
c_{max} = the maximum capacity of each link,
c_a^*, c_a^b = the optimal and basic capacity of link ,
f_p^{rs} = flow on path $p \in P^{rs}$ between demand set (r,s,q^{rs}),
W = the set of origin destination pair (r,s),
$\delta_{a,p}^{rs}$ = a 0, 1 binary variable indicating whether link a is on path $p \in P^{rs}$,

P^{rs} = the set of paths connecting the origin destination pair (r,s),
Z_T = the total travel time objective function,
Z_C = the total construction cost objective function,
Z_E = the environmental impact objective function,
t_a = the travel time of link a, and
x_a = the flow on link a.

As could be seen, this problem is a nonlinear multi-objective problem. In this paper, the problem was converted into a single objective optimization problem using a utility function, as in Eq. (3):

$$U = \frac{Z - Z_{\min}}{Z_{\max} - Z_{\min}} \tag{3}$$

Where $Z_{\max}$, $Z_{\min}$ are the maximum and minimum values of a single objective problem with objective Z, respectively. Thus, the upper level objective function of problem (2) could be written as equation (4):

$$\min_C W_T * \frac{Z_T - Z_{T\min}}{Z_{T\max} - Z_{T\min}} + \tag{4}$$

$$W_C * \frac{Z_C - Z_{C\min}}{Z_{C\max} - Z_{C\min}} +$$

$$W_E * \frac{Z_E - Z_{E\min}}{Z_{E\max} - Z_{E\min}}$$

Where:

(W_T, W_C, W_E) = the set of weight factors which indicate the level of importance of each objective.
Thus, problem (2) could be rewritten as (5).

$$\min_C W_T * \frac{Z_T - Z_{T\min}}{Z_{T\max} - Z_{T\min}} + \tag{5}$$

$$W_C * \frac{Z_C - Z_{C\min}}{Z_{C\max} - Z_{C\min}} +$$

$$W_E * \frac{Z_E - Z_{E\min}}{Z_{E\max} - Z_{E\min}}$$

Subject to:

$$Z_T = \sum_{a \in A} x_a * t_a(c_a, x_a)$$

$$Z_C = \sum_{a \in A} e_a * l_a * (c_a - c_a^b)^2$$

$$Z_E = \sum_{a \in A} (\alpha_0 + \alpha_1 * v_a + \alpha_2 * v_a^2 + \alpha_3 * v_a^3) * x_a * l_a$$

$$c_a^b \le c_a \le c_{\max} \qquad \forall a \in A_p$$

$$\min_X \sum_{a \in A} \int_0^{x_a} t_a(c_a^*, w)\, dw$$

$$\sum_{p \in P^{rs}} f_p^{rs} = q^{rs} \qquad \forall rs \in W$$

Subject to:

$$x_a = \sum_{rs \in W} \sum_{p \in P^{rs}} f_p^{rs} * \delta_{a,p}^{rs}$$

The only new variables used in the model were:
e_a = unit cost of link capacity enhancement,
l_a = the length of link a,
v_a = travel speed on link a,
$\alpha_0, \alpha_1, \alpha_2, \alpha_3$ = the parameters set of the emission function for each link, and
X = the vector of link flows.

It is worth mentioning that the proposed objective function is dimensionless. This property of the objective function made the comparison between different components of the objective function possible. In the proposed problem, any model could be used for formulating the vehicular emission. The model used in problem (5) for vehicular emission was the one used in the VISUM software.

SOLUTION ALGORITHEM

In this paper, the solution algorithm proposed for the bi-level optimization problem consisted of two major steps; determining and forming the objective function and solving the single objective bi-level problem. In order to solve the bi-level optimization problem introduced in Eq. (5), a linear approximation of the lower level was used. The sensitivity information of the linear lower level problem was used as the sub-gradient of the upper one or the master problem in this paper.

As could be seen in Eq. (5), in order to compute the objective function, the minimum and maximum values of different components of the objective function were needed. This information could be determined as follows:

- $Z_{T\min}$: could be computed by fully implementing all the projects. This is the network with the maximum capacity. It should be kept in mind that, in the case of the occurrence of Brasses paradox, this might not be necessarily true and the optimization problem minimizing the total travel time needs to be solved.

- Z_{Tmax}: could be assumed to be the current total travel time of the network without any capacity expansion.
- Z_{Cmin}: The minimum amount of network construction cost is zero
- Z_{Cmax}: The maximum construction cost of the network is simply equal to the maximum budget needed for constructing all the network projects.
- Z_{Emin}, Z_{Emax}: should be computed by optimizing the problem with only environmental objective function.

To solve problem (5), the problem was converted to a Linear Bi-Level (LBL) optimization problem in which the objective functions of both upper and lower level problems were similar. This property was used in the sub-gradient method presented in this paper for solving the equivalent Linearized Network Design Problem (LNDP).

The objective function of the UE assignment model can be noted ψ; thus, $\psi = \sum_{a\in A} \int_0^{x_a} t_a(c_a^*, w)dw$ The first order Taylor approximation of ψ in the k[th] iteration (ψ^k) was given as in (6):

$$\Psi^k = \Psi^{k-1} + \nabla\Psi^{k-1} * (x^{k-1} - x^k) \quad (6)$$

Thus, the objective function of the lower level problem could be written as in (7):

$$\min_X \Psi^k = \min_X [\Psi^{k-1} + \nabla\Psi^{k-1} * (x^{k-1} - x^k)] \quad (7)$$

Subtracting the fix terms from the minimization yields (8):

$$\min_X \Psi^k = \min_X \nabla\Psi^{k-1} * x^k \quad (8)$$

By substitution:

$$\min_X \Psi^k = \min_X \sum_{a\in A} t_a^{k-1} * x_a^k \quad (9)$$

This was similar to the upper level travel time (Z_T) objective function. The term $\frac{\partial t_a}{\partial c_a}$ could be easily calculated by assuming $\frac{\partial x_a}{\partial c_a} = 0$. This assumption is not a very restricting one as, in the linearized equilibrium sub-model, changes in the capacities are small in each iteration and capacity expansion makes the shortest path between each origin-destination pair even more attractive. Thus, assuming that flow does not change with the change in capacity, in each iteration of the algorithm, only has a negligible effect.

The sensitivity information of the linearized lower level problem given in (6) could be easily calculated as:

$$\frac{\partial Z_T^k}{\partial c_a} = \frac{\partial \sum_{a\in A} x_a^k * t_a^{k-1}(c_a, x_a)}{\partial c_a} \quad (10)$$

An important issue about Eq. (10) is that Z_T^k is not smooth; thus, the derivative given by the sensitivity analysis is only one of its sub-gradients, especially when Ψ^k is degenerate. Based on the methodology described in this section, the steps of the solution algorithm could be given as follows:

Step 0- Initiation: set the convergence parameter ε and Set k=1; define the set A_p of candidate links.

Step 1- All or nothing assignment: perform an all or nothing assignment based on $T^k = \{t_a(x_a^{k-1}, c_a^{k-1}) \ \forall a \in A\}$, the vector of link travel times at iteration k.

$$\min_{YX} \sum_{a\in A} t_a(x_a^{k-1}, c_a^{k-1}) * yx_a^k \quad (11)$$

Subject to:

$$\sum_{p\in P^{rs}} g_p^{rs} = q^{rs} \qquad \forall rs \in W$$

$$yx_a^k = \sum_{rs\in W} \sum_{p\in P^{rs}} g_p^{rs} * \delta_{a,p}^{rs} \qquad \forall a \in A$$

In the above equation:

yx_a^k = the auxiliary link flow on link a at iteration k,

YX = the set of auxiliary link flows, and

g_p^{rs} = is the auxiliary path flow on path p between the origin-destination set r and s.

Step 2- Direction finding: Compute the derivative based on Eq. (9) and find the optimal solution of problem (13).

In the above equation:

yc_a^k = is the auxiliary capacity of link a, and

YC = the set of auxiliary link capacities.

Step 3- Line search for the equilibrium assignment: perform the ordinary line search to the exact UE assignment as in the frank-wolf algorithm by solving problem (12).

$$\min_{\gamma}\sum_{a\in A}\int_0^{x_a^{k-1}+\gamma*(yx_a^k-x_a^{k-1})} t_a(c_a^{k-1},w)\,dw \qquad (12)$$

Subject to:

$0 \le \gamma \le 1$

Set $x_a^k := x_a^{k-1} + \gamma * (yx_a^k - x_a^{k-1})$ and go to step 4.

Step 4- Line search for the upper level NDP: in order to find the optimal step size of the upper level problem, the following one-dimensional line search problem should be computed by solving problem (14).

Set $c_a^k := c_a^{k-1} + \beta * (yc_a^k - c_a^{k-1})$ and go to step 5.

Step 5- Check for convergence: if convergence reaches, stop; otherwise, update the travel time and go to Step 1.

RESULTS & DISCUSSIONS

The test network in this study was the Sioux Falls, North Dakota, network, given in Fig. 1 which was first used by LeBlanc. It consisted of 24 nodes and 76 links. 10 links were selected for capacity enhancement, namely, links 16, 17, 19, 20, 25, 26, 29, 39, 48 and 74 in Fig. 1. The maximum capacity enhancement on each link was assumed to be 25 units.

Different combinations of the weight factor were used in order to study the effect of different policies and strategies on the outcome of the proposed model. The travel time function used in this paper was the BPR function as given in Eq. (15):

$$\min_{YC}\left(\frac{W_T}{Z_{T\max}-Z_{T\min}}\right)*\sum_{a\in A}\left(x_a^{k-1}*\frac{\partial t_a(x_a^{k-1},c_a^{k-1})}{\partial c_a}*yc_a^k\right)+ \qquad (13)$$
$$\left(\frac{W_C}{Z_{C\max}-Z_{C\min}}\right)*\sum_{a\in A}\left(2e_a*l_a*(c_a^{k-1}-c_a^b)*yc_a^k\right)+$$
$$\left(\frac{W_E}{Z_{E\max}-Z_{E\min}}\right)*\sum_{a\in A}\left(\left(\alpha_1*\frac{\partial t_a(x_a^{k-1},c_a^{k-1})}{l_a*\partial c_a}+2\alpha_2*v_a*\frac{\partial t_a(x_a^{k-1},c_a^{k-1})}{l_a*\partial c_a}+\right.\right.$$
$$\left.\left.3\alpha_3*v_a^2*\frac{\partial v_a(x_a^{k-1},c_a^{k-1})}{\partial c_a}\right)*yc_a^k\right)*l_a*x_a^{k-1}$$

Subject to:

$$c_a^b \le yc_a^k \le c_{\max} \qquad \forall a \in A_p$$

$$\min_{\beta}\left(\frac{W_T}{Z_{T\max}-Z_{T\min}}\right)*\sum_{a\in A}x_a^k*t_a(x_a^k,\beta)+ \qquad (14)$$
$$\left(\frac{W_C}{Z_{C\max}-Z_{C\min}}\right)*\sum_{a\in A}e_a*l_a*\left((c_a^{k-1}+\beta*(yc_a^k-c_a^{k-1}))-c_a^b\right)^2+$$
$$\left(\frac{W_E}{Z_{E\max}-Z_{E\min}}\right)*\sum_{a\in A}\alpha_1*v_a(x_a^k,(c_a^{k-1}+\beta*(yc_a^k-c_a^{k-1})))+\alpha_2*v_a^2(x_a^k,(c_a^{k-1}+\beta*(yc_a^k-$$
$$c_a^{k-1})))+\alpha_3*v_a^3(x_a^k,(c_a^{k-1}+\beta*(yc_a^k-c_a^{k-1})))*l_a*x_a^k$$

Subject to:

$0 \le \beta \le 1$

$$t_a = t_{0a} * \left(1 + \theta * \left(\frac{x_a}{c_a} \right)^{\mu} \right) \quad (15)$$

For each link a in the network:

t_a = link travel time,
x_a = link flow,
c_a = capacity on link a,
t_{0a} = free flow travel time on link a, and
θ, μ = parameters for calibration (usually set equal to 0.15 and 4 respectively).

In this problem, e_a was set to 0.001 and the unit cost of expansions is given in Table 1.

Table 1. Unit cost of expansion on links

No.	Link	Cost
1	16 and 19	650
2	17 and 20	1000
3	25 and 26	625
4	29 and 48	1200
5	39 and 74	850

As mentioned previously, the emission function used in this study was the function used in the VISUM software. It is worth mentioning that the proposed methodology was not sensitive to the function used for evaluating emission. The parameters used in this example network were as follows:

$\alpha_0 = 16.425$
$\alpha_1 = -0.38357$
$\alpha_2 = 0.0028706$
$\alpha_3 = -0.0000045425$

The first step towards the solution of the problem was to compute the minimum and maximum for each single objective problem. The results are given in Table 2.

Table 2. Maximum and minimum of each single level problem

Objective	Minimum value	Maximum value	Difference
Z_T	78.546	113.626	0.308732
Z_C	0	4325	1
Z_E	8945.511	9634.161	0.07148

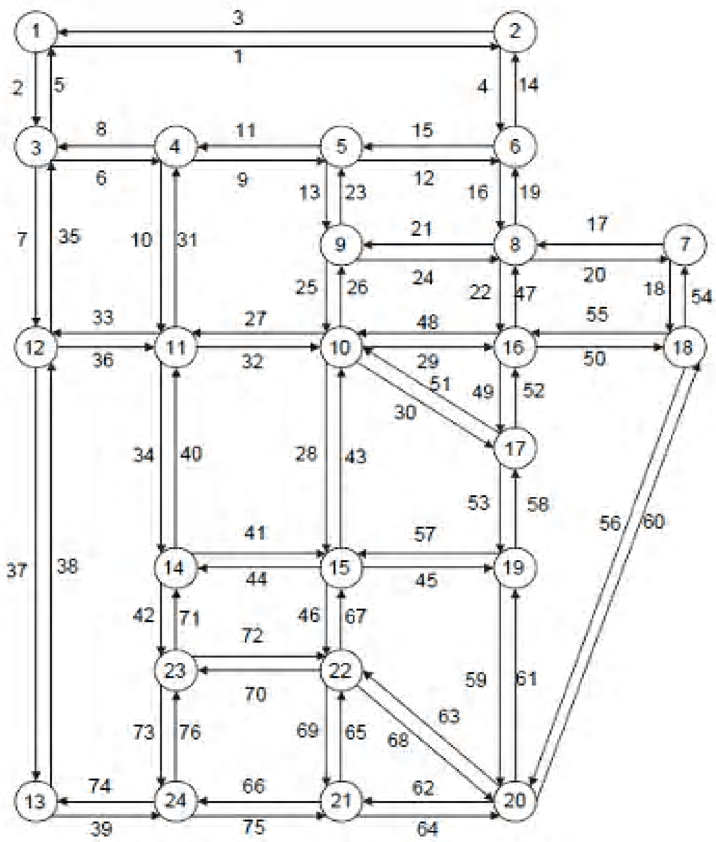

Fig. 1. The Sioux Falls network

Table 3. The different weight factors used in this study

Weights	Case 1	Case 2	Case 3	Case 4
W_T	0.25	0.25	0.5	0.33
W_C	0.25	0.5	0.25	0.33
W_E	0.5	0.25	0.25	0.33

Table 4. The capacity expansion of each candidate link in the network

Link	Case 1	Case 2	Case 3	Case 4
6.8	24.91	17.071	24.067	22.241
7.8	13.11	7.291	10.459	9.824
8.6	24.91	17.071	24.067	22.241
8.7	13.11	7.291	10.459	9.824
9.10	18.838	9.274	16.286	14.089
10.9	18.838	9.274	16.286	14.089
10.16	25	18.723	24.972	24.848
13.24	24.91	16.518	24.767	22.829
16.10	25	18.723	24.972	24.848
24.13	24.91	16.518	24.767	22.829

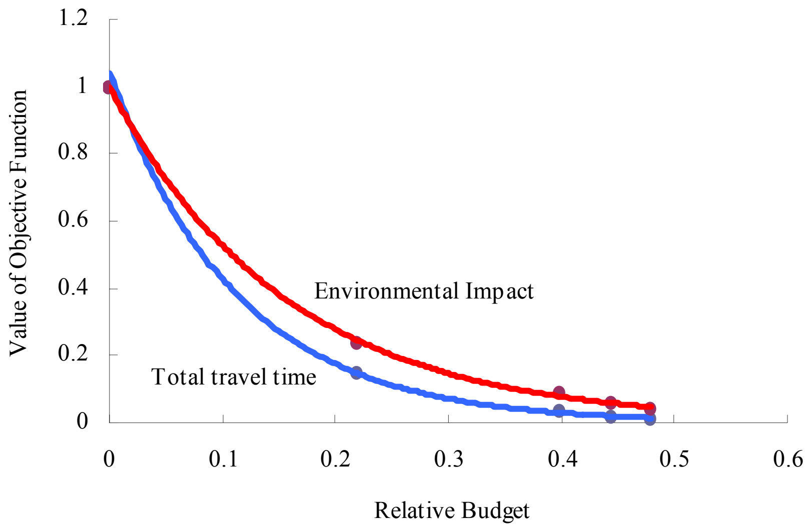

Fig. 2. The value of each part of objective function with respect to budget

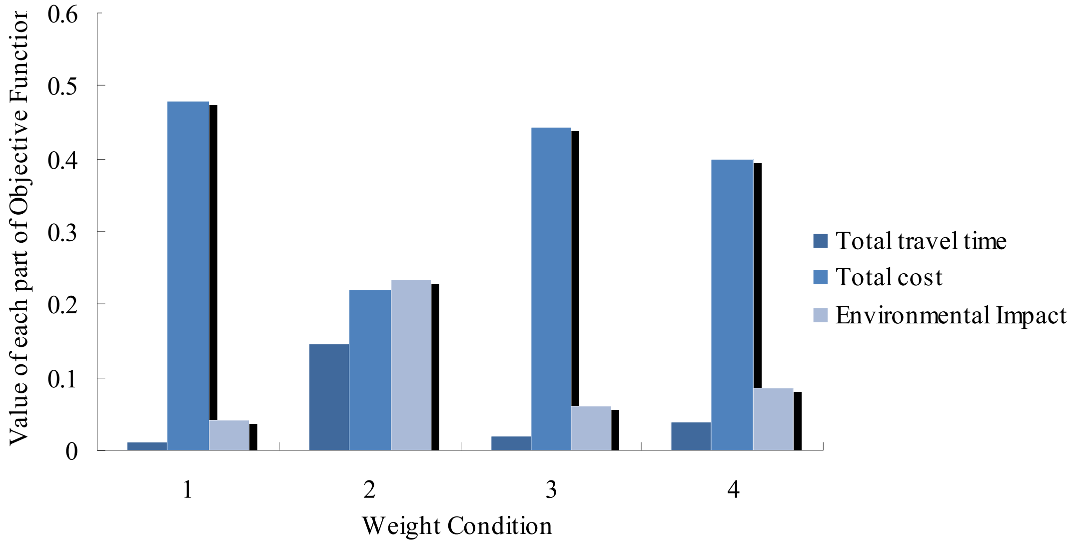

Fig. 3. The value of each part of objective function in each weight condition

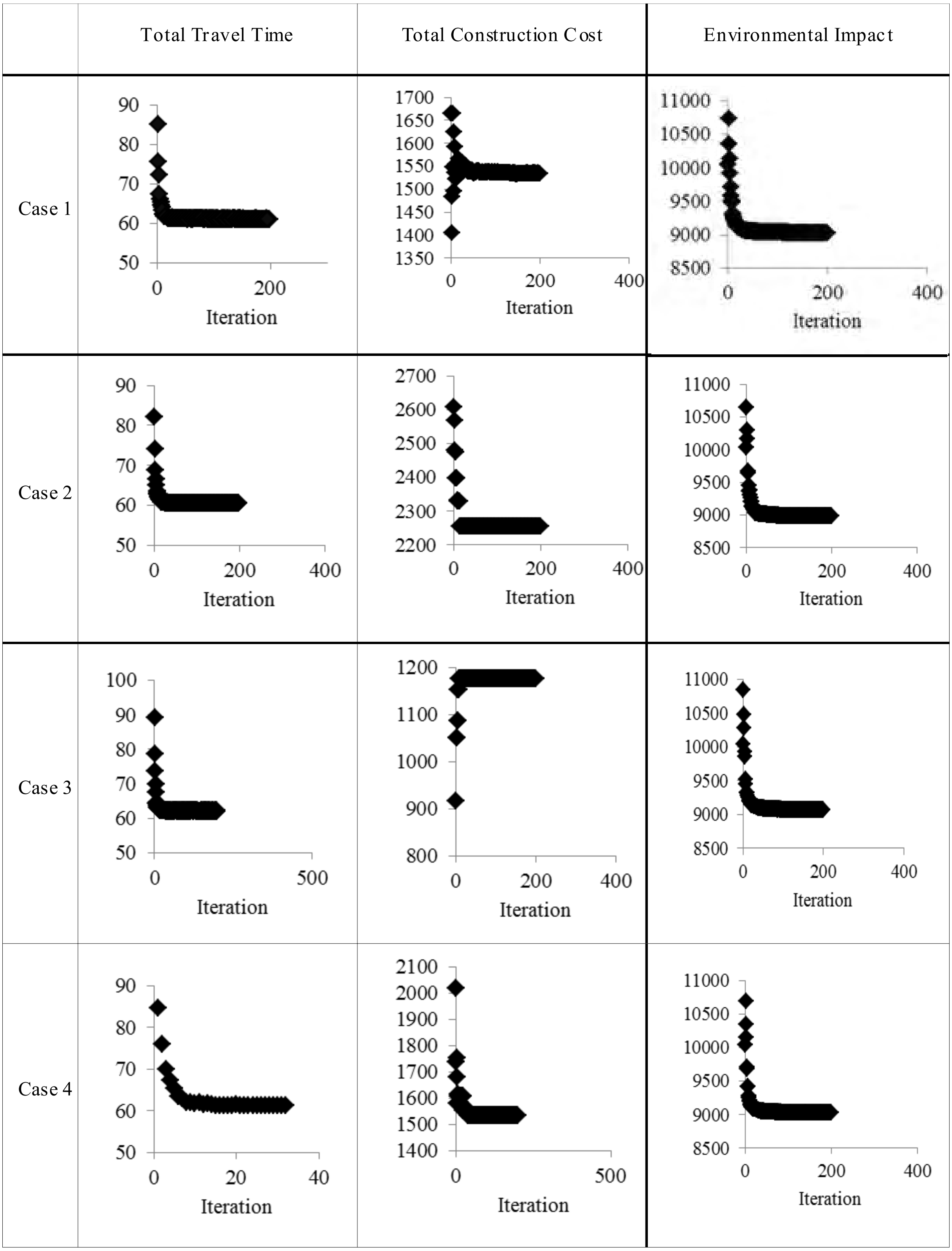

Fig. 4. The convergence of different part of objective function in each weightingCombination

The Solution methodology given in this paper was implemented in the GAMS software. Four different weighting conditions were used as presented in Table 3 in order to estimate the effect of different policies on the objective function.

With respect to Table 4, it could be seen that different weight combinations result in different link capacity improvement. These differences were substantial in some cases, which showed the importance of different policy decisions on capacity enhancement. For instance, the capacity expansion in case 1 for link 6.8 was equal to 24.91 while, in case 2, it was equal to 17.071.

These radical changes could be seen in other links as well. There is no clear-cut solution for obtaining the optimal weight in these multi-objective problems although methods such as the so-called Simple Average Weighting (SAW) or Analytic Hierarchical Process (AHP) could be used for obtaining an estimate of the weights.

Another measure which could be used for estimating the effectiveness of a given weight combination is the value of each part of objective function in problem (5) as shown in Fig. 2. In this fig. based on maximum possible budget level, budget has been calculated relatively.

Fig. 3 shows the value of each part of objective function in problem (5) in each weighting condition. As could be seen the relative change between weight condition 1 and 3 are insignificant which shows that a 7 percent increase in budget level doesn't substantially improve the travel time and environmental condition of the network.

The convergence of each part of objective function in each weighting combinations are given in Fig. 4. As could be seen in this fig the convergence in all the objectives is obtained in just a few iterations and increasing the number of iterations doesn't substantially improve the objective function further. This could be used as a measure of convergence of the algorithm and an indication of achievement of the optimal value.

CONCLUSION

Capacity enhancement is one of the major issues in urban areas and Network Design Problem (NDP) deal with this important subject in transportation planning. The Continuous Network Design Problem (CNDP) has been traditionally formulated as a bi-level optimization problem. In this paper, the CNDP was modeled as a multi-objective linear bi-level optimization problem, and the problem was solved by a partial linearization scheme in order to easily calculate the sensitivity information of the lower level problem. The multi-objective master problem was converted to a single objective problem using weighted utility measures. These dimensionless utilities were tested for different weight combinations in order to show the effect of different policies and decisions on different objective values. It was shown that the proposed methodology was efficient for implementing realistic size networks. Inclusion of the vehicular emission to the objective function could be mentioned as another innovation of this paper. The implementation of the proposed method or a meta-heuristic method to a real size large network could be considered for future research.

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