

## Statistical and Dynamic Modelling of Algae in Stream Systems

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**ABSTRACT:** Algal growth and the water quality effects were studied and modeled in the lower reaches of the Colusa Basin Drain as a stream system. Statistical Techniques such as the correlation matrix and the stepwise multiple regression are performed on such variables as algal biomass, ammonia, nitrate, orthophosphate, temperature, and the solar radiation intensity in deriving an algae statistical model. Orthophosphate is shown as the limiting nutrient controlling the growth kinetics of algae. A one-dimensional dynamic mathematical and computer model is formulated to simulate suspended algae and water quality parameters. The mathematical solution is obtained by a finite-difference implicit method. The model is calibrated and confirmed with the data collected from the lower 30 km reaches of the Colusa Basin Drain. The simulation results for both the statistical and the dynamic models are in good agreement with measured values. The model can be used to evaluate water quality in stream systems.

**Key words:** Algae, Statistical, Dynamic, Modeling, Water Quality, Streams, Limiting Factors

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### INTRODUCTION

Analysis of suspended algae in response to limiting factors has been useful in understanding algal growth dynamics, because it has frequently revealed causes for change in population density (Colterman, 1975; Droop 1974; Rhee, 1978.) Population density is controlled by: 1) physical factors such as light intensity, temperature, turbidity, and turbulence; 2) biochemical factors such as inorganic nutrients, organic nutrients, growth-promoting and inhibiting substances, toxic materials, and heavy metals; and 3) biological factors such as parasitism, predation, and competition. In the ideal situation one factor is so rate limiting that it controls the algal biomass while all other factors may be at optimal levels.

This paper attempts to relate suspended algal biomass to changes in limiting nutrients and physical factors. Because population changes in nature can rarely be explained by only one factor, it is necessary to use methods of analysis that permit evaluation of several factors simultaneously. Statistical techniques such as

multiple regression analysis make possible the simultaneous consideration of several suspected factors and the evaluation of relative importance of each factor. In addition to the empirical-statistical techniques, a dynamic simulation approach can be developed on the basis of conservation of mass to describe and predict the behavior of suspended algae and nutrient in the system.

Although excellent references are available in discussions of algae and nutrient in lakes (Chen, 1970 and Ditoro *et al.*, 1970), less is known about algae and nutrient in stream systems. The objectives of this study are:

a) To determine to what extent the algal biomass abundance and growth kinetics could be correlated with limiting nutrient concentrations and physical factors; and b) to develop and compare the statistical and dynamic models which approximate the suspended algal biomass and water quality parameters in the prototype.

## MATERIALS & METHODS

The study area is the Colusa Basin Drain (CBD) that is located on the west side of the Sacramento Valley. The CBD conveys flood runoffs and irrigation waters from about 4,050 km<sup>2</sup> of watershed and agricultural lands into the Sacramento River. Water samples for biological and chemical analysis were obtained from CBD and two tributaries; Stone Corral at Two Mile Road and Glenn-Colusa Irrigation District (GCID) main canal. Samples were taken at the surface, one secchi-disc depth and two secchi-disc depth with a Van Dorn bottle cast. Chemical parameters measured were: total organic carbon (carbon analyzer), dissolved oxygen (membrane electrode), ammonia nitrogen (calorimetric phenolate), nitrate nitrogen (calorimetric), organic nitrogen (total kjeldahl), and orthophosphate phosphorus (calorimetric). Temperature was determined with a bucket thermometer. Light availability for photosynthetic organisms was measured in the water column using a LI-185 quantum/radiometer/photometer; algal biomass was determined by measuring chlorophyll-*a*. Assuming 1.5 percent of the dry weight of organic matter (ash-free weight) of the algae is chlorophyll-*a*, algal biomass may be estimated by multiplying the chlorophyll-*a* content by a factor of 67. Chlorophyll-*a* was measured using the fluorometric technique suggested by Vollenweider (1963). Magnesium carbonate (MgCO<sub>3</sub>) was added as a preservative to prevent acidification of the acetone. Under acidic conditions chlorophyll can degrade into phaeophytin and other physiologically inactive products. These products have absorption peaks in the same regions of the spectrum as the chlorophyll and produce errors in chlorophyll determination. These potential errors were minimized by addition of MgCO<sub>3</sub>, but some error may still exist as a result of the presence of phaeophytin and other chlorophyll degradation products among the living cells. In addition to the above, hydraulic parameters of the flow regime were also determined. The correlation matrix on the data revealed that except for solar radiation intensity and temperature, other variables were not correlated significantly. The solar radiation and temperature were correlated, with  $r^2$ -values greater than 0.70. Generally, if  $r^2 = 0.7$  for two variables, the variable that shows greater

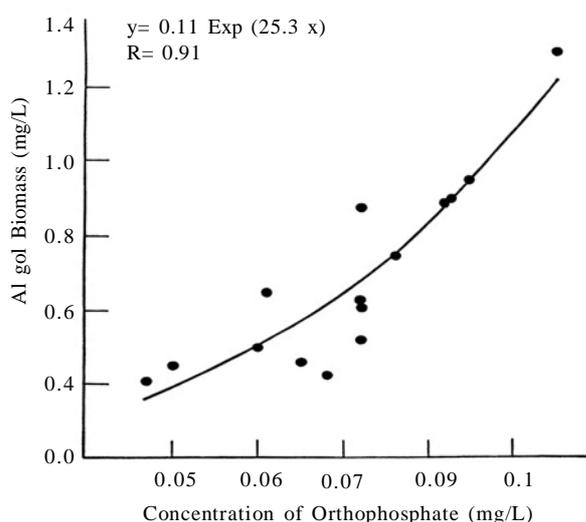
correlation should be considered for further analysis. In the present study, the solar radiation showed a negative correlation  $r^2 = -0.1$  with algal biomass, so temperature, with a  $r^2$  value of 0.060, was used as independent variable for further analysis. The results of the stepwise multiple regression analysis on algal biomass and on variables such as orthophosphate, nitrate, ammonia, and temperature revealed the order of importance of the variable. These variables accounted for 99 percent of the variation of suspended algae in the study area. Orthophosphate accounted for 70 percent, and the other two variables for 29 percent. The results of multiple regression analysis led to the development of regression model given as:

$$G_{SA} = 12.34 PO_4 - 0.015 T + 1.06 NO_3 + 0.12 NH_3 - 0.046$$

In which  $G_{SA}$  = suspended algal biomass (mg/L),  $PO_4$  = orthophosphate concentration (mg/L),  $NO_3$  = nitrate concentration (mg/L),  $NH_3$  = ammonia concentration (mg/L), and T = Temperature (°C). Phosphorus in the Colusa Basin Drain water appears to be the limiting nutrient controlling algal biomass productivity. The phosphorus concentration was low, ranging from 0.01 to 0.10 Mg/L, and the sources of phosphorus are most probably from the chemical weathering of primary minerals and decomposition of soil organic matter and vegetation. Phosphorus compounds in soils in the form of calcium, aluminum, or iron. Phosphates usually have very low solubility and hence phosphorus concentration levels in water are typically low. Phosphorus is also adsorbed by clay and organic particles and these particulate forms dominated in CBD. Since the current velocity of the CBD dropped from upstream to downstream, CBD-3 (Colusa Basin Drain at Tule Road) to CBD-1 (Colusa Basin Drain at Road 99E and 108), the suspended particle settled out. Therefore, a decrease in phosphate concentration in this region of the CBD was due to phosphate adsorption by suspended particles (Tanji *et al.*, 1980).

The results of this analysis between algae and orthophosphate indicate that the algal biomass is exponentially related to the orthophosphate concentration, as shown in Fig. 1. A strong

correlation coefficient  $r^2 = 0.91$  indicates that the orthophosphate is the most important factor controlling algal growth in the study area. To develop a relation between algal biomass and the limiting factor a simple regression analysis was performed.



**Fig.1. Relationship between algal biomass and orthophosphate**

level  $t_j$ . Statistical methods were used to relate the algal biomass to factors controlling its growth kinetics. These methods are described below:

1. Correlation Matrix Method- This method was performed to determine which of the variables (ammonia, nitrate, orthophosphate, temperature and solar radiation intensity) are related to algal biomass. This method singles out the highly correlated variables. Algal biomass was treated as a dependent variable, and all other factors were treated as independent variables.

2. Stepwise Multiple Regression Method- After determining a correlation matrix, stepwise multiple regressions was performed to evaluate which independent variables explained the variation in the dependent variable, the dependent variable was algal biomass, and independent variables were orthophosphate, nitrate, ammonia, and temperature. For this analysis, an updated version of the statistical package for the social sciences (SPSS) was used. Two main criteria were applied at each step as a basis for selecting the best equation describing the algal biomass productivity in the system.

a) The F-test was used to determine the

significance of the regression and to evaluate whether the amount of variation in the algal biomass described by the regression equation was statistically significant.

b) The square of the multiple correlation coefficients ( $r^2$ ) was compared with the standard error of the estimate before and after each new independent variable was added to the equation.

3. Simple Regression- After the correlation matrix and multiple regression analysis were performed on the dependent and independent variables, simple regression analysis was carried out to develop a functional relation between the algal biomass and its limiting factor.

### Dynamic Simulation Approach

In general, dynamic simulation modeling of phytoplankton and nutrients is at an advance stage of development. These models are based on conservation of mass and they take into account the hydrodynamic transport and the internal sources and sinks (Chen *et al.*, 1976, Contractor *et al.*, 1980, Ditoro *et al.*, 1973 and Toerien *et al.*, 1973). The majority of these models have been applied to real systems with satisfactory results. The major differences between the various models are the form of internal sources and sinks. Some models include predation of phytoplankton by zooplankton (Chen *et al.* 1970 and Ditoro *et al.*, 1973) and such factors as temperature, light intensity and nutrient.

A dynamic simulation model was developed to describe and predict the behavior of suspended algae and nutrient. The model calculates the growth and decay of algae in the drain or river system in response to flow rate, water temperature, solar radiation intensity, and nutrient concentration.

### Model Formulation

The formulation is based on the mass balance for a particular substance. The statement of the mass balance can be given as:

$$\text{Accumulation} = \text{Inflow} - \text{outflow} + \text{sources or sinks}$$

The mass-balance statement yields the equations for the suspended algae, ammonia, nitrate, and orthophosphate. These equations take into account advective transport, diffusive

transport, growth and respiration, chemical reactions, and lateral inflow of point and non-point sources. The equations are written in one dimensional form, assuming steady-state, non-uniform, spatially varied stream flow, as follows:

1. Mass Balance Equation for Suspended Algae:

$$\frac{\partial S_a}{\partial t} = -\frac{1}{A} \frac{\partial}{\partial X} (AuS_a) + \frac{1}{A} \frac{\partial}{\partial X} [AD_x \frac{\partial S_a}{\partial X} + \frac{1}{A} \frac{dS_a}{dX} + (\mu - r - S)S_a] \quad (2)$$

In which  $S_a$  = concentration of suspended algae (mg/L),  $A$  = cross-sectional area of flow ( $m^2$ ),  $u$  = average current velocity (m/sec),  $t$  = Time (sec),  $\mu$  = local specific growth rate of algae (L/day),  $r$  = local respiration rate of algae (L/day), and  $S$  = local settling rate (m/day). The third term in equation 2 takes into account the lateral inflow or outflow  $Q_x S_a$  in which  $Q_x$  = flow rate ( $m^3 / sec$ ).

2. Mass-Balance Equation for Nitrogen

$$\frac{\partial N_1}{\partial t} = -\frac{1}{A} \frac{\partial}{\partial X} (AUN_1) + \frac{1}{A} \frac{\partial}{\partial X} [AD_x \frac{\partial N_1}{\partial X} + \frac{1}{A} \frac{dN_1}{dX} + (\alpha_1 r S_a - \beta_1 N_1 + \alpha_2 / A) - \alpha_3 S_a] \quad (3)$$

In which  $N_1$  = concentration of ammonia nitrogen (mg/L),  $\alpha_1$  = fraction of respired algae re-solubilized as ammonia nitrogen (mg/L),  $\alpha_2$  = benthos source rate for ammonia nitrogen (mg/L.day),  $\alpha_3$  = fraction of algal biomass that is ammonia (mg/L),  $\beta_1$  = rate constant for the biological oxidation of ammonia. The third term in equation 3 takes into account the lateral inflow ( $Q_x N_1$ ).

$$\frac{\partial N_2}{\partial t} = -\frac{1}{A} \frac{\partial}{\partial X} (AUN_2) + \frac{1}{A} \frac{\partial}{\partial X} (AD_x \frac{\partial N_2}{\partial X}) + \frac{1}{A} \frac{dN_2}{dX} + (\beta_1 N_1 - \alpha_1 U S_a) - \beta_2 S_a \quad (4)$$

In which  $N_2$  = concentration of nitrate nitrogen (mg/L) and  $2$  = fraction of algal biomass i.e. nitrate (mg/L).

3. Mass Balance Equation for phosphorus

$$\frac{\partial P}{\partial t} = -\frac{1}{A} \frac{\partial}{\partial X} (AUP) + \frac{1}{A} \frac{\partial}{\partial X} (AD_x \frac{\partial P}{\partial X}) + \frac{1}{A} \frac{dP}{dX} + (\alpha_4 r S_a - \alpha_4 \mu S_a - \alpha_4 \mu S_a + r / a) \quad (5)$$

In which  $P$  = concentration of orthophosphate ion (mg/L),  $\alpha_4$  = fraction of algal biomass that is phosphorus and  $r$  = benthos source rate for phosphorus (mg/L.day). The third term in equation 5 takes into account the lateral inflow ( $Q_x P$ ).

### Method of Solution

An implicit finite difference scheme was used to solve the above partial differential equations numerically. This scheme has the advantages of being stable even when large time intervals are chosen, which, in turn, provides considerable savings for long-term simulation of large-scale problems. To apply the solution procedure, equations 2 through 5 were written in the following generalized form:

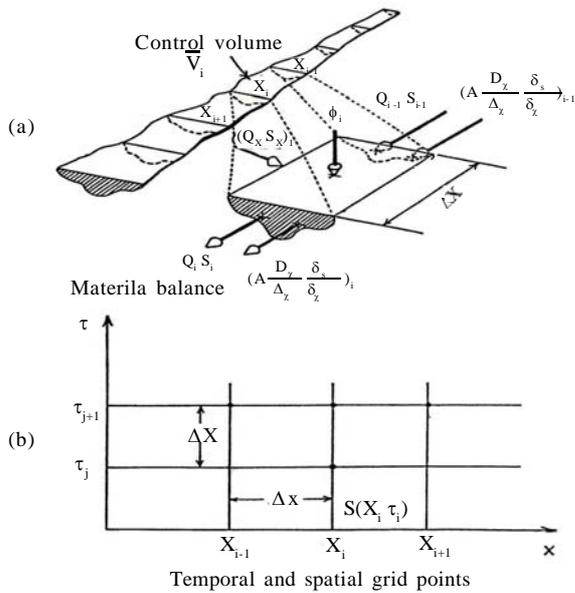
$$\frac{\partial S}{\partial t} = -\frac{1}{A} \frac{\partial}{\partial X} (QS) + \frac{1}{A} \frac{\partial}{\partial X} (AD_x \frac{\partial S}{\partial X}) + \phi \quad (6)$$

In which  $S$  = concentration of constituent to be simulated,  $Q$  = flow rate, and  $\phi$  = sources and sinks. The equation was written in an implicit finite difference form for reach length  $x$  and grid points  $I-1, I, I+1$ , as shown in Fig. 2a.

The temporal derivative was approximated for a finite set of coordinates  $X$  and  $t$ . the spatial derivatives were approximated along three points,  $I-1, I$ , and  $I+1$  at time  $J+1$  as shown in Fig. 3b. All variables were known for all computational elements of the network at the time level  $t$ . The known values of the variables at time were found by solving the system of linear algebraic equations formulated from tridiagonal matrix which resulted from substitution of the finite-difference approximations. The special technique used for solving such matrix equations was the modified Gaussian elimination algorithm. The equations were solved by taking proper boundary and initial conditions. Details of the solution procedure are found in documentation (Mirbagheri *et al.*, 1981).

### Initial and Boundary Condition

The concentration of suspended algae and nutrients along the CBD was not known at the beginning of the simulation period. Therefore, in the present study, the initial conditions are said to be zero. Since the transport is unidirectional in most freshwater streams, i. e., there is no significant transport upstream, the concentration in the flow entering the CBD-3 was taken as the upstream boundary condition,  $S_0^{J+1}$ .

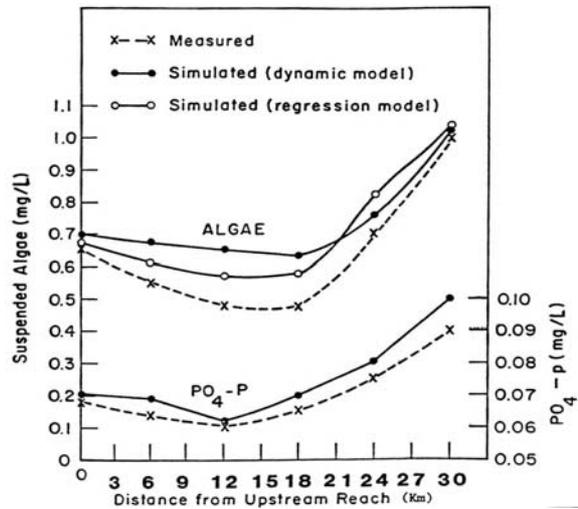


**Fig. 2. Discretized Stream System (After Water Resources Engineers, Inc., 1976)**

Since the ration  $x/uT$  in the numerical scheme should be approximately equal to the current velocity of water in the prototype, suspended algae and nutrient will travel a distance  $x$  in a time interval  $t$ . Thus if only advection operates, the downstream boundary condition can be given approximately by  $S_{N+1}^{J+1} = S_N^J$  where  $S_{N+1}^{J+1}$  is the concentration just downstream from the end of the system i.e., CBD-1.

**Simulation Results**

Measured and simulated concentration profiles along the Colusa Basin Drain were compared to determine the accuracy of the models. The suspended algae concentration was simulated by both the regression analysis and the dynamic models as shown in Fig. 3. Both models followed basically the same pattern as the measured value of suspended algae. Such behavior was expected, because phosphorus was considered to be a limiting nutrient. In the regression model, the concentration of orthophosphate controls algal biomass and light has no effect, whereas the dynamic model considers the effect of both light intensity in the water and the concentration of orthophosphate. The algae concentration simulated by the regression model is closer to the measured value than is the value simulated by the dynamic model. The regression model over estimates the value of suspended algae from CBD-1A (km 24) to CBD-1 (km 30), as shown in Fig. 4 possibly because of a high concentration of nitrate.



**Fig. 3. Algal and Phosphorus Simulation Results**

Generally, the algal biomass should increase with time and distance, but it decrease between CBD-2A (6 km) and CBD-1B (18 km) and then increased. This may be done to endogenous respiration and to the settling rate of algae. The lateral inflow of algae form the rice fields between CBD-1B and CBD-1 is another possible explanation. Orthophosphate, ammonia, and nitrate were simulated by the dynamic model, as shown in Fig. 3 and 4. Since the concentration of phosphorus and algae are interrelated, the concentration profile for phosphorus followed the same pattern as algae. Figure 4 shows that the agreement between simulated and measured values of phosphorus is satisfactory. The simulation results of ammonia followed the same pattern as the measured one within 80% agreement, as shown in Fig.4. Simulated data on nitrate overprotected the concentration of nitrate form CBD1A to CBD-1, as shown in Fig. 4. A large population of terrestrial and aquatic plants grow on the bank and in the water, but nitrate uptake by there plants is not considered in the model, for no data are available on the biomass of these plants. In general, the simulated results for suspended algae and nutrients agreed well with the measured values, the overall behavior of this system is reasonably well represented by the model output.

**CONCLUSION**

The principal objective of this research was to obtain a more clear understanding of algal rowth

and limiting nutrients as well as transport mechanisms in fully mixed stream systems. A regression model was developed based on the correlation matrix and stepwise multiple-regression analysis on such variables as algal biomass, ammonia, nitrate, orthophosphate, temperature, and solar radiation intensity. The specific growth, endogenous respiration, settling rate of algae, and half-saturation constant of phosphorus were evaluated. The rating curves for the current velocity, discharge, and water depth were established. The functional relation were developed and used as a flow model to generate hydraulic data for the dynamic model. Based on the model formulation, a computer code was developed to simulate suspended algae and nutrient in the one-dimensional fully mixed Colusa Basin Drain. The model was calibrated and validated based on the data collected along the Colusa Basin Drain.

Finally from the study of algae and the limiting nutrient we conclude that:

1. Phosphorus is the most important limiting nutrient.
2. Nitrate and temperature are of secondary importance.
3. Photo inhibition is significant at the water surface at high light intensity.
4. A simple statistical model agrees somewhat with kinetic models on algal growth mainly controlled by phosphorus.
5. The statistical model simulates closer to measured values than the dynamic model but it is site-specific.

6. The simulated dynamic model agrees well with the measured values.
7. The dynamic model can be applied to other systems and other locations if provided the necessary input information.
8. Both models can be applied to stream systems to evaluate water quality parameters.

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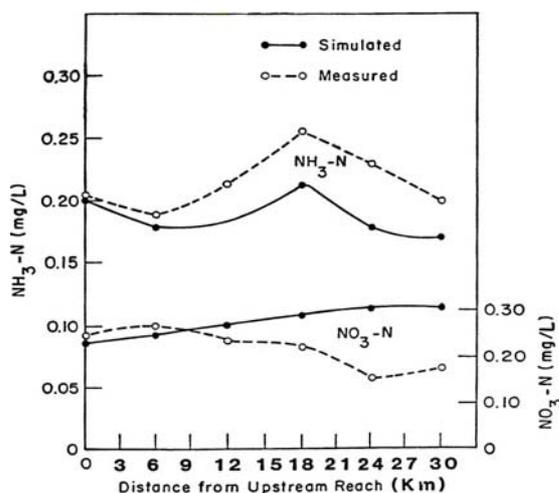
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**Fig. 4. Ammonia and Nitrate Simulation Results**